AN APPLICATION OF TIME-STEP SIMULATION TO ESTIMATE AIR DEFENSE SITE SURVIVABILITY

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SUMMARY

This research presents a time-step simulation methodology that rapidly generates accurate estimates of air defense site survivability when an integrated missile defense is attacked by more than one enemy aircraft. A comparison and brief review of the major simulations used to study air-to-ground warfare is included.

The simulation presented, called AIRDEF, allows the user to specify the number of estimates to be generated, the number of battles to be executed in the generation of each estimate, and the size of the time-step to be used. The aircraft are modeled in terms of their position, speed, number, and engagement range of ground targets; the air defense sites are modeled in terms of their number, position, acquisition range, tracking and firing minimum times, target acquisition capability, missile speed, and missile launch and kill probabilities. The user inputs the aircraft and air defense parameters once for initialization. The demonstration of AIRDEF includes a typical printout from a representative time step, a complete execution of the simulation, and the results of an example run of simulation experiments.

The methodology and simulation presented were developed for use by analysts studying air-to-ground combat. Anticipating that many estimates of site survivability will be needed by tactical commanders over a narrow range of attack aircraft parameters, a methodology for efficiently summarizing these estimates in a simple regression equation is also presented and demonstrated. The equation for tactical use is obtained by generating three estimates for each cell of a 3^4 experiment which varies aircraft number, speed, attack range, and electronic counter-measure capability, using ANOVA and regression techniques to formulate a predictive equation valid for the conditions of interest.

The research concludes that AIRDEF and the associated methodology produces accurate and useful estimates of air defense site survivabilities. It also provides evidence that time-step simulation may be more fruitful than next-event simulation in further studies of the same type.

CHAPTER I

INTRODUCTION

Estimation of survival probabilities in engagements is crucial in the tactical employment of military assets, in the design of new military assets, and in strategic military planning. Viability of the United States nuclear strategy, in which any United States nuclear strike force will consist of survivors of a preemptive attack by an opposing power, is particularly sensitive to the accuracy and reliability of survival probability estimations.

In the particular area of engagements between attacking aircraft and ground-emplaced air defense missile sites, rapid technological progress provides a continuing challenge to the study of survivability. Recent technological advancements in air power include radar homing missiles, electronic counter measures (ECM) to frustrate detection by air defense radar, terrain-following navigation systems to allow aircraft to fly below minimal heights for effective ground-mounted air defense radar, air-to-surface weaponry of increasing range and firepower, and sophisticated on-board electronic equipment to monitor air defense radar activity.

To counter these advances, new air defense systems are under continuing development. The U.S. Army currently has at least two major air defense development efforts underway. In addition, tactical redeployments and minor asset improvements (such as dummy radar tracking stations, and improved Hawk missiles, for example) are constantly being proposed, reviewed and implemented. The decision criteria guiding these developments, redeployments and improvements can be viewed as striking a balance between time and cost considerations on one hand and survivability considerations on the other.

1.1 Objective and Procedure

The primary objective of this research is to develop a usable simulation methodology for estimating air defense missile site survivability probability, with emphasis on flexibility and generality of application to a wide range of battle conditions, when the site is deployed in conjunction with other air defense missile sites in an integrated defense. A secondary goal is to develop simple equations to predict air defense site survivability probabilities for narrower ranges of battle conditions, based on regression of data collected in simulation experiments using the full methodology. The investigation consists of a review of the currently available attempts to estimate air defense site survivability probability (ADSSP), the development of a computer simulation program to estimate ADSSP, the generation of ADSSP estimates for use in an analysis of variance (ANOVA), and the use of the generated ADSSP estimates and the results of the ANOVA in step-wise regression analysis to produce a predictive equation for ADSSP.

1.2 Review of ADSSP Estimate Literature

An analytical method for determining the survivability of a single air defense site when attacked by more than one aircraft was published on a limited basis by the Air Warfare Division, United States Army Materiel Systems Analysis Agency, Aberdeen Proving Grounds, Maryland, in June, 1974 (8). This work was done by Ronald A. Halahan and was intended only as an informal account of an interim nature; it was distributed to a few relevant U.S. Army agencies. Halahan used the concept of an expected value model utilizing the binomial probability distribution to represent the expected probabilities of occurrence. In general, the Halahan model is as follows:

$$EP_s = \sum_{i=1}^{N_i} P_i P_s(i) \qquad \text{, where}$$

EP_s = the expected probability of site survival,

N_i = the number of ways the site can interact with the attacking aircraft,

P_i = the probability of occurrence of the ith interaction,

$P_s(i)$ = the probability of site survival for the ith interaction.

The Halahan model has been computerized by AMSAA and requires a relatively small amount of user input prior to execution. The obvious shortcomings of this model are that it deals with only one unsupported air defense site, and that it requires as input survivability estimates $P_s(i)$ that are unavailable except through simulation or direct experience. These shortcomings led directly to the sponsorship of the present research by the Air Warfare Division, AMSAA.

No other analytical work was found for either single or multiple air defense site survivability. The inappropriateness of the analytical approach is suggested by the mathematical complexity of the Halahan model, which contains four separate interaction possibilities, each with at least three distinct probability terms. The addition of more air defense sites in support of the site of interest would lead to a multinomial probability distribution instead of a binomial distribution and would enlarge the Halahan model to such an extent that it would be far too complex to solve analytically.

There have been several attempts to investigate ADSSP through the use of next-event computer simulations of air-to-ground warfare. Robert E. Shannon developed a next-event simulator in 1968 for the United States Army Missile Command (MICOM), Redstone Arsenal, Huntsville, Alabama (15). The Shannon simulator, like the Halahan model, considers only one site. Shannon further restricted his efforts to only one attacking aircraft firing air-to-ground missiles. This simulation, with the one-on-one combatant restriction, is a low-resolution simulator which attempts to determine only single ADSSP estimates and can execute 1000 simulated air-to-ground engagements in 166 seconds on a UNIVAC 1108 computer. This is in marked contrast to the high-resolution simulations in use by the United States Army today.

The most widely known high-resolution air defense simulation program is use today is TACOS (Tactical Air Defense Computer Simulation). This, like the Shannon simulator, is a next-event simulation. TACOS is documented in an executive summary and 17 separate programmer/analyst manuals. This simulation package is so detailed that its practical use on a day-to-day basis has proved impossible. Execution time along for some runs of TACOS has taken up to 12 hours. TACOS was designed not to give ADSSP estimates, but to be used at a much higher level for testing concepts of tactical deployment and determining multi-system performance characteristics. Currently, in order to determine a single estimate of ADSSP it is necessary to examine the entire TACOS print-out manually, an extremely time consuming task for large scale TACOS runs. Tens or scores of such single estimates would be required to develop a reliable ADSSP estimate for one set of conditions. TACOS does have some very desirable features -- it can handle multiple attackers and defenders, it allows for a mixture of weapons systems, and it can include terrain analysis -- but the extensive set-up and run time preclude routine use for developing ADSSP estimates.

Two other high-resolution simulations currently in use in the study of air-to-ground warfare are relevant. MADS (Missile Air Defense Simulation) is similar to TACOS except that it handles only one type air defense system at a time (14). It is somewhat less time-consuming than TACOS, but it fails to address ADSSP estimation directly. TAGWAR (Tactical Air-Ground Warfare Simulation) is being utilized at The Rodman Laboratory, Rock Island Arsenal, Rock Island, Illinois (16). This simulation employs the same basic logic as TACOS except for the Monte Carlo aspect of statistics development. TAGWAR generates probability trees for each possible aircraft-to-air defense site relationship. As the simulation proceeds through the battle, event to event, the appropriate probability tree is traversed. Precomputed probabilities of aircraft survivability are accumulated for use in the output, which is a time-based probability of aircraft survivability. It would appear that this program could be modified to reflect ADSSP estimates, but this would require a large amount of reprogramming. Additionally, the time spend in developing the probability trees for a large scale battle make this program too costly in time and money for daily use. Also, like the Halahan model, it demands input data that properly should be output.

The salient features of the currently available tools for investigation of air defense systems are summarized in Table 1.

Work was done in 1973 by Dr. N. N. Prui for the U.S. Army Missile Command (MICOM) to find a means of reducing the execution times of the MICOM simulation programs and still maintain high resolution (14). Puri's proposed solution was for a time-increment-advanced simulation as opposed to the current next-event methods. He hypothesized that time-interval-advanced would reduce the time spend in TACOS and MADS for the generation of terrain data and event lists. The suggestion of time-increment simulation is consistent with general recommendations in the simulation literature (6,15) regarding the choice simulation methods. The present work is intended not only to provide an improved method for generating ADSSP estimates, but also to demonstrate the superiority of the time-increment approach in simulating this type of battle and to provide a framework on which to build more efficient replacements for more comprehensive simulations such as TACOS.

Table 1. Comparisons of Existing Air Defense Models

	Model				
Attributes	Halahan	Shannon	TACOS	MADS	TAGWAR
Implementation	Computer	Computer	Computer	Computer	Computer
Technique	Analytic	NE Siml	NE Siml	NE Siml	NE Siml
Combatants	M-on-1	1-on-1	M-on-N	M-on-N	M-on-N
Weapons Mix	AC only	No	Both	AC only	Both
Resolution	High	Low	High	High	High
Preparation Time	Low	Low	High	High	High
Execution Time	Low	Low	High	High	High
Daily Use	Yes	Yes	No	No	No
ADSSP Directly	Yes	Yes	No	No	No

CHAPTER II

DEVELOPMENT OF THE METHODOLOGY

2.1 Introduction

As was seen in the literature of estimation of air defense site survivability probability (ADSSP), all existing methods capable of handling multiple combatants are next-event simulations. Their high resolution power is overshadowed by high set-up costs and execution times that preclude simulation runs long enough to obtain reliable ADSSP estimates. For a realistic level of detail in modeling battles involving multiple combatants on each side, the problem is to reduce the set-up and execution time of the required simulation experiments. Following the suggestions of Puri (14), the present research employs the time-interval-advance simulation technique to this end.

2.2 Underlying Assumptions

To develop a simulation program of sufficient detail and still have relatively short set-up and execution times the following assumptions are imposed on the aircraft vs. air defense sites scenario:

2.2.1 Assumptions Regarding Attacking Aircraft

1. Electronic counter measure (ECM) capabilities possessed by the aircraft are modeled in terms of a degradation of the air defense sites' target acquisition capabilities.

2. Aircraft velocity and direction remain constant, though not necessarily equal for all aircraft, throughout the simulation.

3. Aircraft possess anti-air defense suppression ordnance, including radar homing missiles of varying range capability and kill probability.

4. Aircraft possess sufficient electronic capability to acquire and engage all air defense sites within range of their anti-air defense weapons.

5. Aircraft firing doctrine will be to fire on all air defense sites within range of their anti-air defense weapons.

The practical impact of the above aircraft assumptions is to present an attacking force that knows the location of all defense sites within their attack range and will fire on all of these know sites, modeled in terms of the number, position, speed, and engagement range of these attacking aircraft.

2.2.2 Assumptions Regarding Air Defense Sites

1. All air defense site locations remain constant and within mutual support distance.

2. Upon successful acquisition of an aircraft, air defense sites must track the aircraft for some minimum time before attempting to engage the aircraft.

3. Air defense sites possess an unlimited supply of constant-velocity missiles which will be fired at the closest unengaged aircraft within range.

4. Should a successfully launched missile fail to kill an aircraft the air defense site will immediately attempt to refire on that aircraft.

5. Should an attempted launch be unsuccessful, the air defense site must wait some minimum time before attempting to fire at any aircraft.

The practical impact of these air defense site assumptions is to present a mutually supportive system of air defense sites which can track all incoming aircraft but can fire on only one aircraft per site at any given time, modeled in terms of their number, location, acquisition range, tracking and firing minimum times, target acquisition capability, missile speed, and missile launch and kill probabilities.

2.2.3 Battle Environmental Assumptions

1. The effects of weather, terrain, and time of day play no part in the simulated battle.

2. ADSSP is not affected by any other enemy forces, such as indirect firing field artillery, direct fire tanks, or sabotage.

2.3 Simulation Limitations

The assumptions utilized in this research restrict the universal applicability of the simulation in its present form. The following conditions are not directly modeled by this simulation:

1. the effects of terrain masking of aircraft;
2. aircraft which change flight path and direction randomly;
3. air defense sites which can control more than one missile at a time.

The limitations of the simulation force it to represent a "high altitude" air defense system, for example a NIKE-HERCULES air defense system. This representation allows terrain effects, "diving attacks" and the air defense problem of slew rates to be ignored.

2.4 The Simulation

Events in the system defined by the above assumptions depend critically on interrelationships among continuously changing variables (aircraft and missile positions). Time-interval-advanced simulation is generally recognized as superior to next-event simulation for systems having this characteristic (6,7,15). The underlying logic of time-advanced simulation is to examine the entire system at regular time intervals and to keep track of all changes which occurred during the elapsed time.

The time-advanced simulation developed herein, called AIRDEF, investigates an aircraft to determine which air defense sites it has killed, updates the surviving air defense sites with respect to their interrelationships with this aircraft, and proceeds to the next aircraft; when this has been done for all aircraft, the aircraft are moved forward, the simulated battle time is incremented, and the procedure is repeated. See Figure 1 below.

INCREMENT TIME

CHECK FIRST AIRCRAFT

UPDATE ALL AIR DEFENSE SITES

CHECK NEXT AIRCRAFT

.
.
.

MOVE ALL AIRCRAFT

INCREMENT TIME

Figure 1. Overall AIRDEF Battle Logic

A more detailed logic structure of AIRDEF is presented in the following text and in the flowcharts for Figures 2 through Figure 8. Appendix A contains the complete Fortran code for AIRDEF.

AIRDEF is started with the simulated battle time equal to 0. The initialization portion of each AIRDEF battle collects data defining the conditions of the simulation and performs transforming calculations for future use. The initializing data given by the user are:

1. the number, position (in AIRDEF conversion coordinates) and speed (in knots) of each attacking aircraft;

2. the number and position (in AIRDEF conversion coordinates of each air defense site;

3. the speed (in ground speed knots) of the aircraft and the air defense missiles;

4. the minimum delay time (in % of a minute) between refiring attempts;

5. the minimum tracking time (in % of a minute) for each air defense site;

6. the maximum air defense acquisition range (in kilometers);

7. the time increment to be used to advance the battle time (in % of a minute);

8. the minimum effective range of aircraft weapons (in kilometers), and the weapons' kill probability (in %);

9. the number of air defense radar sweeps per minute the user desires the acquisition probability to be built upon.

10. the air defense probabilities (in %) for aircraft acquisition, launch, and intercept;

11. a seed for a uniform random number generator if the computer time clock is not used for this purpose.

Based upon the user supplied data above AIRDEF automatically performs the following initializations prior to execution of the battles:

1. generates and saves for each battle to be run 500 random numbers from a uniform (0,1) distribution;

2. transforms all velocities from knots to kilometers per minute;

3. translates the AIRDEF converted coordinates for all aircraft vs. air defense sites into a single dimension (X). This step enables the computation of the Y-Z plane range for each set of combatants only once so that future movement and slant range determination will involve only the X dimension.

4. determines the probability of acquiring aircraft at each time step. This is necessary because the number of time steps per minute may be different from the number of radar sweeps per minute. When an aircraft is within acquisition range AIRDEF attempts to acquire it on each time step; when there are more time steps per minute than radar sweeps per minute the acquisition probability for each time step must be decreased. AIRDEF computes the probability of aircraft acquisition for each time step as follows:

$$P_{APTS} = 1 - (1 - P_{APRS})^{[(\Delta t)(RSPM)]} ,$$

where

P_{APTS} = Probability of acquisition per time step,

P_{APRS} = Probability of acquisition per radar sweep

Δt = time step used (% of a minute),

RSPM = Radar sweeps per minute.

Of course, if there are fewer time steps per minute than radar sweeps per minute, the probability of acquisition on each time step must be increased. The above formula is valid for either case.

After initialization AIRDEF checks to insure that the battle has not exceeded the time limit placed upon it. This time limit is provided to stop the simulation after all aircraft are past all air defense sites. If the battle time limit has been exceeded AIRDEF updates the alive-killed status of all combatants and starts a new battle. If the battle time limit has not expired AIRDEF begins the procedure illustrated at node 2 of Figure 2.

For the first aircraft, (AC(1)), the slant range to the first air defense site, (AD(1)), is computed and checked to determine if AC(1) is close enough to AD(1) to kill it. If the slant range is too great AIRDEF moves to the next air defense site. If AC(1) is within range of AD(1) a random number is drawn from the 500 initially generated, compared to the kill probability for AC(1)'s weapons, and AD(1) is eliminated from further consideration during the battle if it was killed. When this has been done for all air defense sites AIRDEF initiates the procedure illustrated in Figure 3.

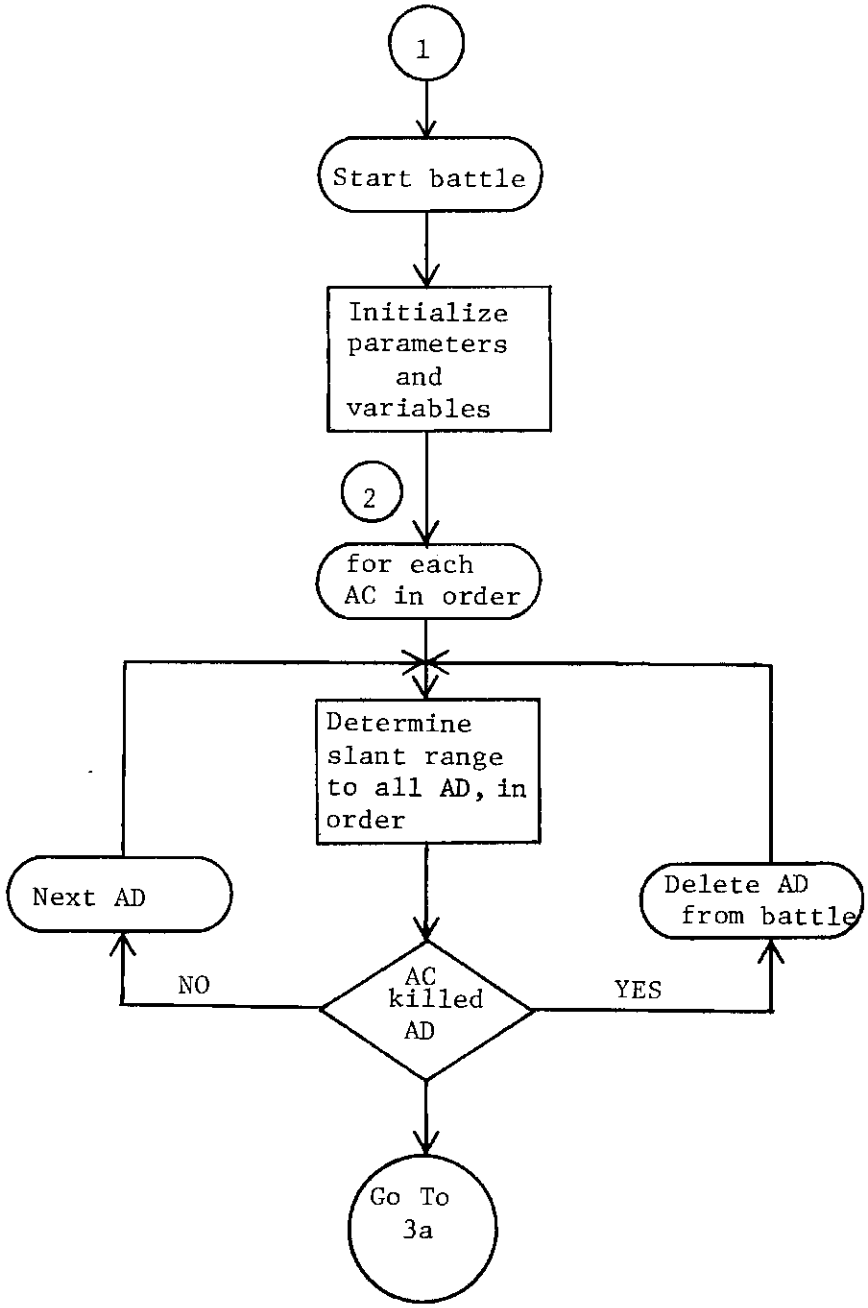

Figure 2. Flowchart for Aircraft Kill of Air Defense Site

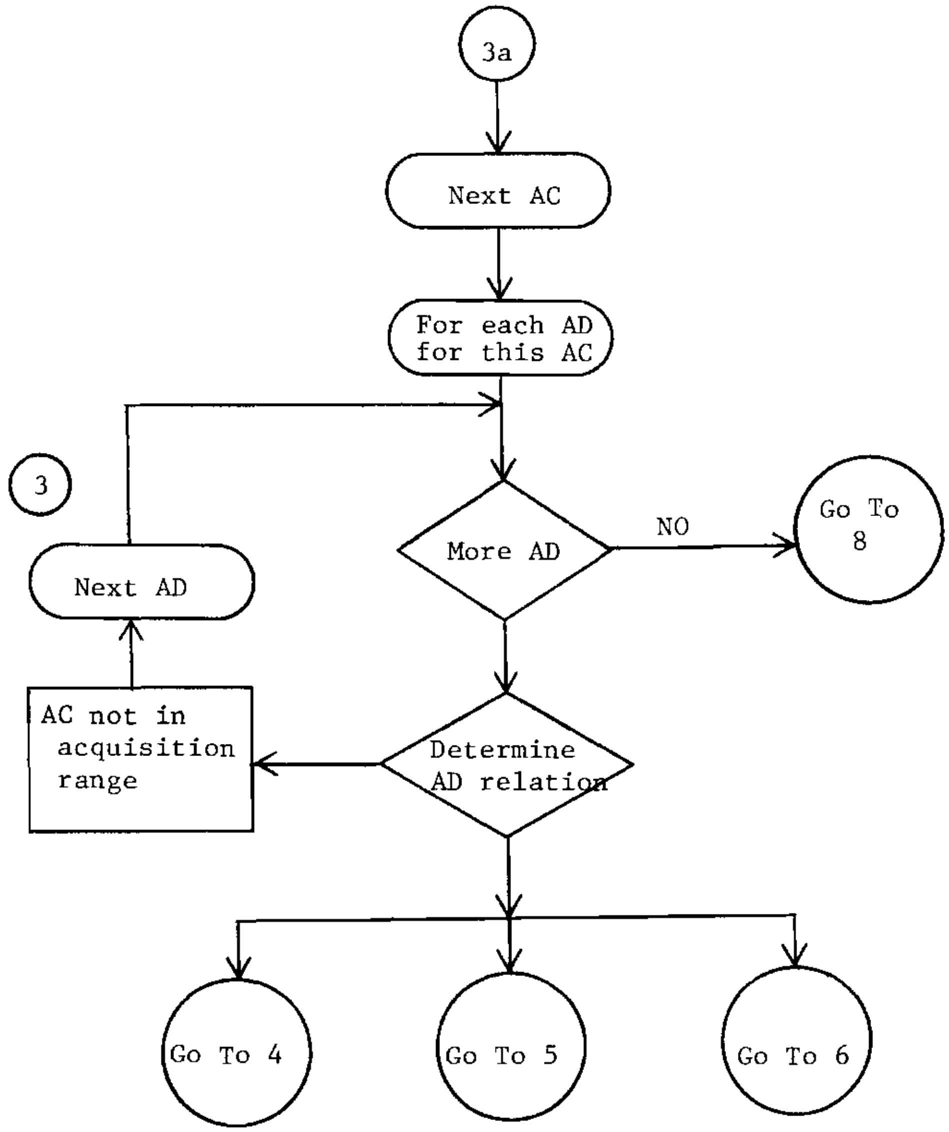

Figure 3. AIRDEF Flowchart for Air Defense Site Relationships

Using the slant range between AC(1) and all surviving sites, AIRDEF begins with AD(1), assuming AD(1) has not been previously killed, and determines what interrelationship exists. There are four possible interrelationships and related site actions which must be determined for AD(1):

1. AC(1) is not within the maximum radar acquisition range of AD(1). The next surviving site is then considered;

2. AD(1) has a missile flying toward AC(1). AIRDEF determines if a kill of AC(1) should have taken place during the elapsed time interval by comparing the intercept time of AD(1)'s missile with the battle time. If intercept should not have taken place AIRDEF will return AD(1) to this interrelationship next time period. If an intercept should have taken place AIRDEF compares a random number with AD(1)'s missile kill probability to determine the result of the intercept. If AC(1) was killed it is eliminated from the rest of the battle and AIRDEF moves to the next site; if AC(1) was not killed, AD(1) immediately attempts to launch another missile. See Figure 4. If the follow-up launch is successful AIRDEF will return AD(1) to the engagement interrelationship next time period; if the launch is unsuccessful AD(1) will continue to track AC(1) during the next time period. See Figure 5.

3. AC(1) is within the radar acquisition range of AD(1), but has not yet been acquired. AIRDEF determines if AC(1) is acquired by AD(1) through the comparison of a uniform random number with the acquisition probability of AD(1), and an elapsed tracking time clock is started if AD(1) acquires AC(1). If AD(1) fails to acquire AC(1), AIRDEF will return to this interrelationship for AC(1), vs. AD(1) next time period.

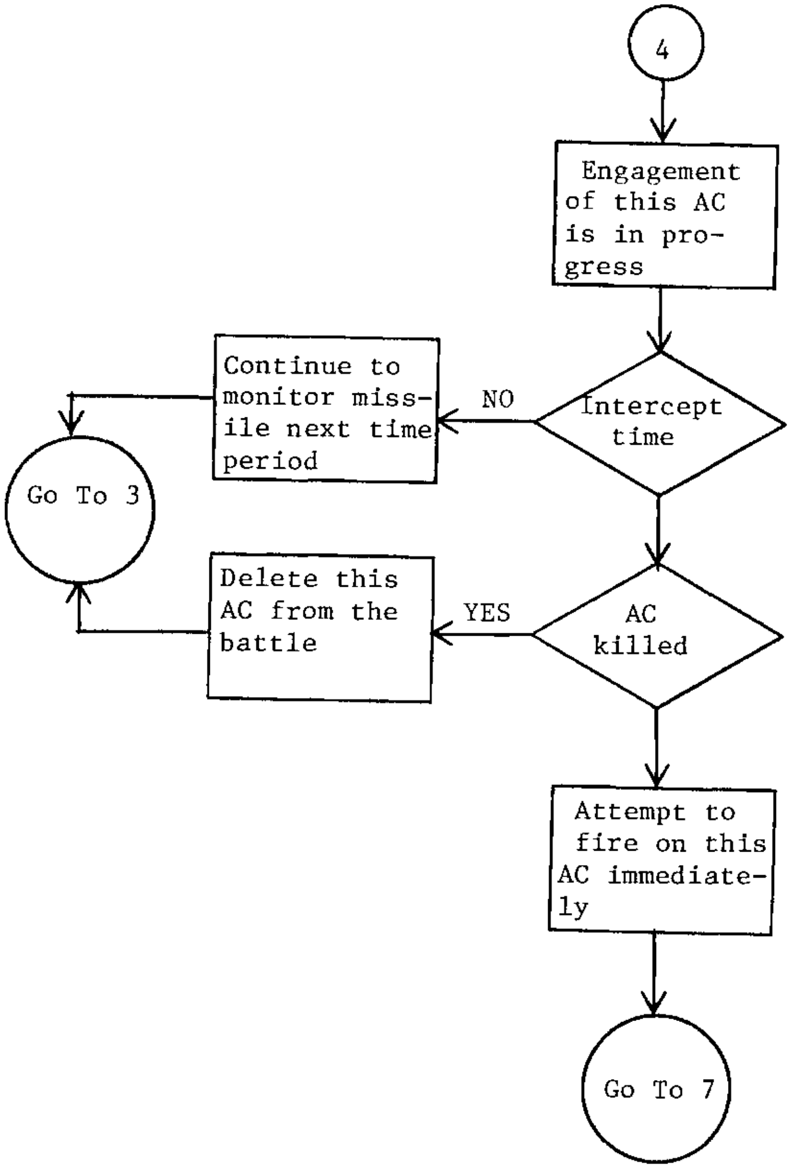

Figure 4. Aircraft Engagement Flowchart

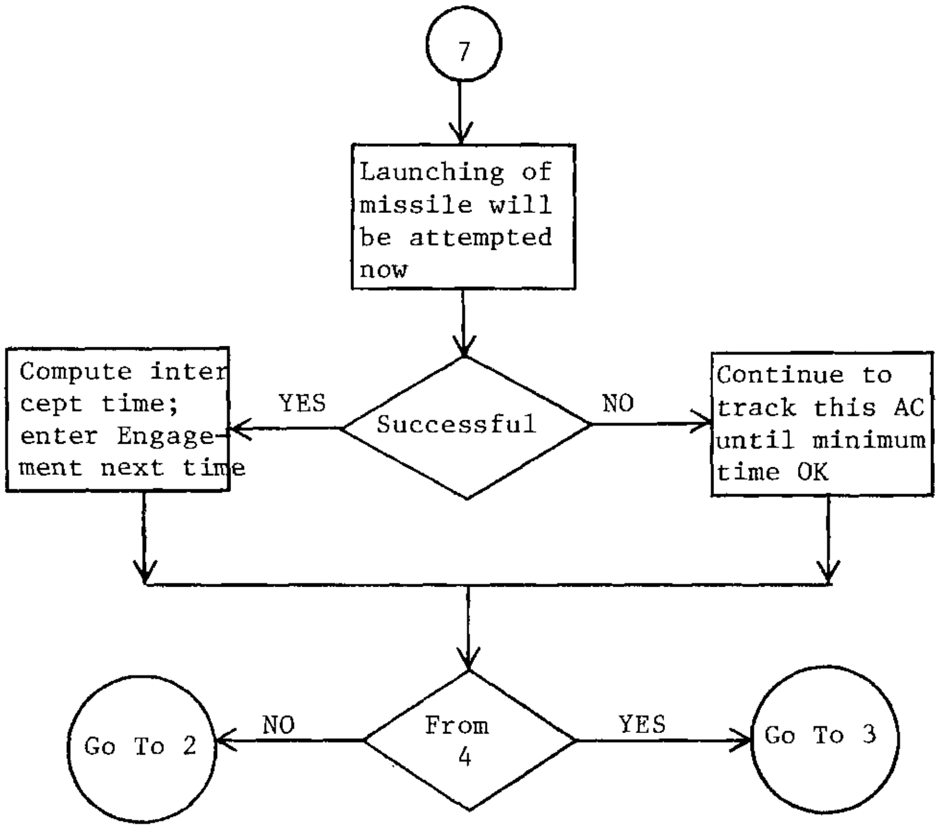

Figure 5. Missile Launching Flowchart

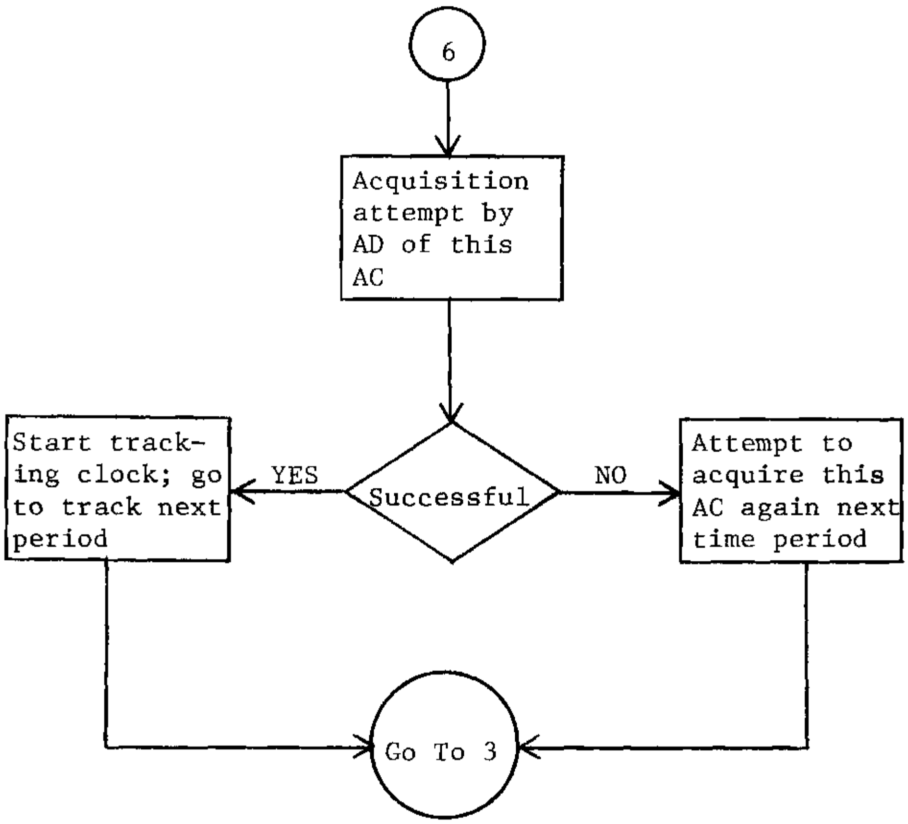

Figure 6. Aircraft Acquisition Flowchart

See Figure 6.

4. AC(1) has been acquired by AD(1) and is currently being tracked. AIRDEF will check the elapsed tracking time clock and if AD(1) has tracked AC(1) long enough AIRDEF will indicate that AD(1) is ready to fire on AC(1). If AD(1) has not tracked AC(1) for the minimum required tracking time AIRDEF will return to this interrelationship for AC(1) vs. AD(1) next time period. See Figure 7.

AIRDEF now cycles through all remaining air defense sites to determine their interrelationship with AC(1). When this is completed AIRDEF looks at all sites to determine which sites are ready to fire on AC(1), and selects from these sites the closest air defense site which is ready to fire and not currently engaging any aircraft. See Figure 8. The launching procedure discussed in interrelationship 2 above and seen in Figure 5 is followed.

AIRDEF now moves to the next surviving aircraft and the entire sequence is repeated until all aircraft have been examined. AIRDEF then determines if there are surviving combatants on each side, and if so moves the surviving aircraft, increments the battle time, and begins the aircraft vs. air defense process again with the first surviving aircraft. If either side has no survivors, AIRDEF updates the alive-killed status of all combatants to reflect the results of the current battle before returning to the initialization portion for a new battle.

Special attention should be given to the tracking and launching portions of AIRDEF (Figure 7 and Figure 5). In the tracking section a check must be made to insure that the air defense site has been tracking the aircraft for some minimum specified time. This requirements is placed

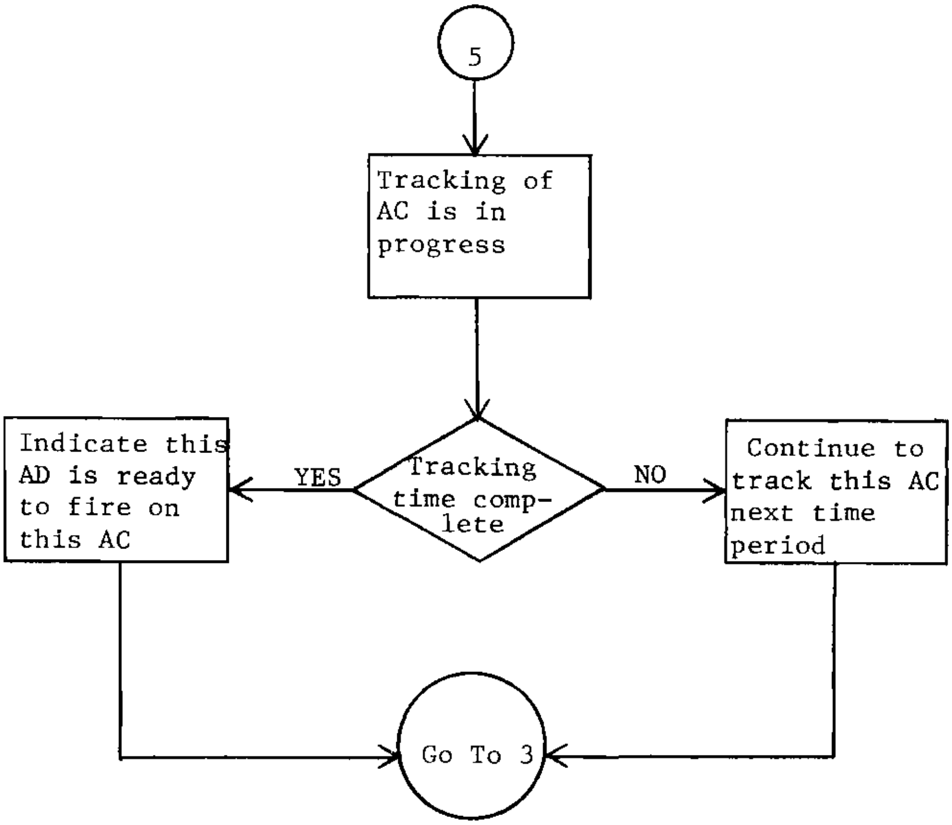

Figure 7. Tracking Time Indicator Flowchart

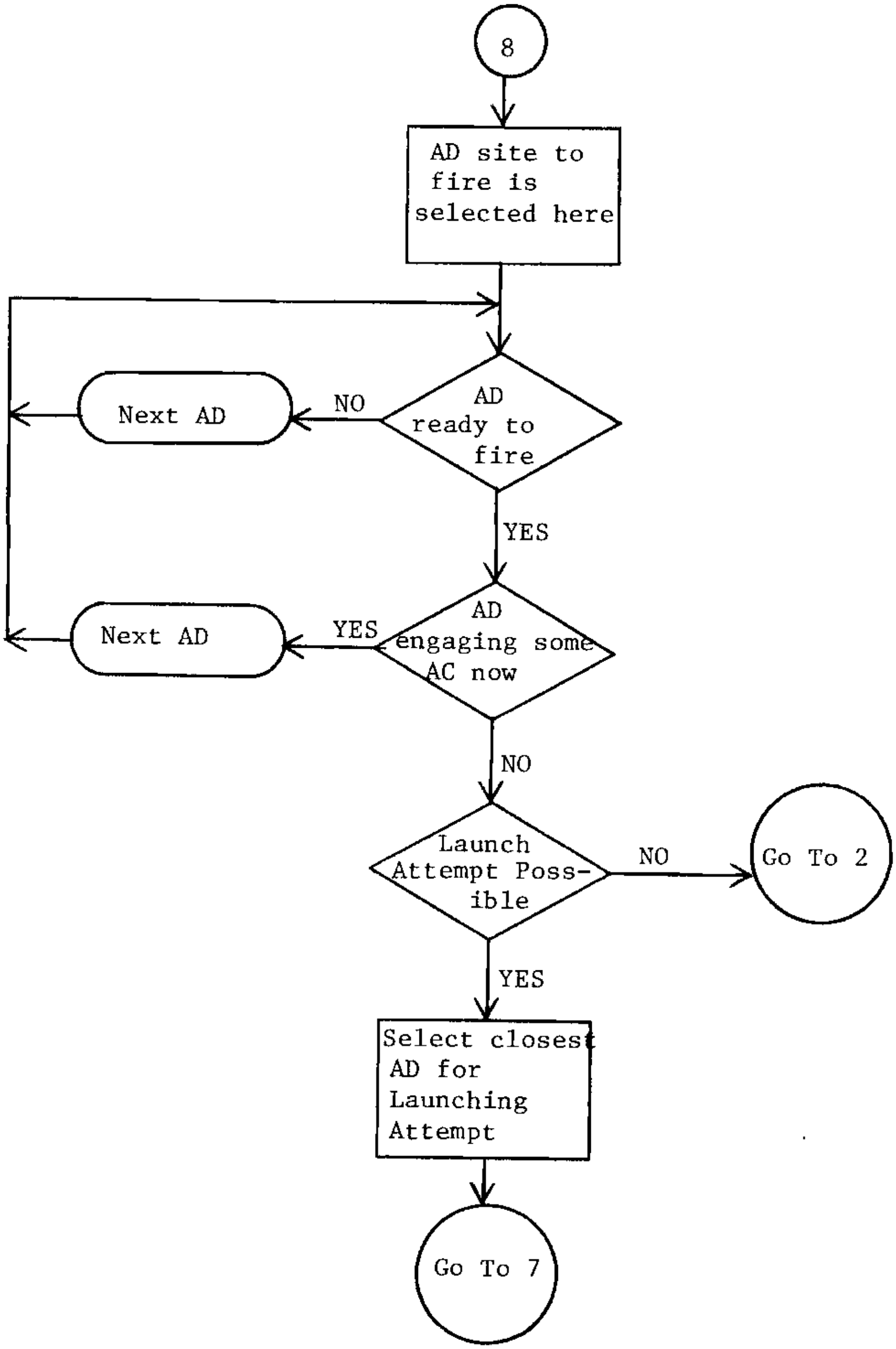

Figure 8. Firing Site Selection Flowchart

upon the air defense site to insure that a true acquisition has taken place and to allow sufficient time to prepare a missile for launch. This restriction may also be viewed as an indicator of the level of training of the air defense site personnel. Well trained personnel should be able to acquire targets and prepare to fire more quickly than less well trained personnel.

In the launching section two outcomes are possible, success or failure of the missile launching attempt. This reflects the reliability of the air defense system and accounts for the rare time when a missile fails on the launching pad. Should a missile launching attempt prove unsuccessful the air defense site must wait the minimum delay time specified by the user in the initialization phase. This time is required in order to prepare another missile for launch, or to repair the faulty missile.

For a successful missile launching the simulation time that the missile should intercept the aircraft must be computed. AIRDEF determines this by dividing the aircraft-to-side slant range by the sum of the aircraft and air defense missile velocities. Obviously, this procedure is correct only when the aircraft is approaching the site and intercept will occur while the aircraft is still approaching. This situation is the one normally encountered in air-to-ground combat of the type simulated by AIRDEF and is the method used throughout AIRDEF for intercept time determination.

The decision rule of looking at the aircraft first is arbitrary and will have some effect on the estimates of ADSSP generated providing the time increment used is not sufficiently small ($\leq$ 0.10 seconds).

Large time increments, say 10 or 15 seconds, will allow the aircraft to fire upon and kill an air defense site after the site should have killed the aircraft. The decision as to the size of the time increment must be made by the user, realizing that smaller time increments will increase the running time of the simulation, but will also decrease the error of considering one combatant before the other, and vice versa.

2.5 Information Accumulation and Statistics Generation

Upon completion of a simulated battle each air defense site is either sill in existance or it has been killed. If the simulated battle is run again with different random numbers a new final status of combatants will result. This set of combatant results is completely independent of all preceding battles. If this process is repeated, say η times, there will be η sets of combatant results, either alive or killed. In AIRDEF this procedure is followed by having the status of each air defense site (and each aircraft) alive at the conclusion of a battle set equal to 1; for each site (and aircraft) killed it is set equal to 0. The variables SUMAD (i), where i=1,2,...,K and K is the number of air defense sites simulated, sum the status of each air defense site for the η repetitions of AIRDEF. After η simulated battles an air defense site, say i, will have SUMAD (i) equal to the number of times air defense site i was not killed. The distribution of their sum is binomial with the parameter p being the probability of surviving a battle (10,12); p is estimated for a given site by determining $\hat{p}(i)$, where

$$\hat{p}(i) = \text{SUMAD}(i)/n \qquad , \; i = 1,2,\ldots,K \text{ sites.}$$

In AIRDEF the estimate of ADSSP for each site is given by the above calculation and is called AVEAD(i), i=1,2,...,K sites. SUMAC(j), j=1,2,...,J aircraft is also collected but not reported.

The question of how large n should be for each estimate of ADSSP must be answered by the user during the interactive phase of AIRDEF. The size of n is generally influenced by two factors, the amount of computer time the user is willing to consume and the confidence level that the user desires to place on the width of the interval of estimation associated with each AIRDEF estimate. From the normal approximation to the binomial distribution (13), the following $1 - \alpha$ large-sample confidence interval for p is obtained:

$$p = \mathrm{AVEAD} \pm Z_{1-\alpha}\left[\frac{(\mathrm{AVEAD})(1-\mathrm{AVEAD})}{n}\right]^{1/2} \tag{1}$$

where $Z_{1-\alpha}$ is the value of a standard (0,1) variable at the $1-\alpha$ percent significance level. This expression when solved for n gives

$$n = \frac{Z_{1-\alpha}^{\;2}(\mathrm{AVEAD})(1-\mathrm{AVEAD})}{(\text{size of error of estimation})^2} \tag{2}$$

Prior to utilizing the ADSSP estimates computed by AVEAD(i) the direction of attack must be considered. AIRDEF executes all n battles with the same spatial arrangement of aircraft vs. air defense sites. If it is known for certain that the only attack possible is from one single unchanging direction, then the estimates AVEAD(i) are correct for each i.

Normally what is known is a "likely direction of attack", or that an attack "is equally likely in all directions." AIRDEF assumes that the aircraft attack is equally likely from all directions and computes the average of all ADSSP estimates. This value, called PSYS, in addition to the individual ADSSP estimates, is printed as part of AIRDEF output. For any other likely direction of attack an expected likelihood modification to AIRDEF will have to be made. This is easily done by modifying AIRDEF such that the estimate for a single site is equal to the sum of the estimates of all sites weighted by the probability that the attack comes from the direction of that site.

If more than one estimate of ADSSP is desired, AIRDEF is executed more times with n battles run each time. This is incorporated into AIRDEF by having the user specify the value of IN during the initialization phase. The number of battles used to estimate ADSSP is also inputted as the value of LIMIT. For example, AIRDEF executed with IN equal to 3 and LIMIT equal to 75 will produce three estimates of ADSSP, each based upon 75 simulated battles. Utilizing equation (1) or (2) above with α equal to 0.05, one can be 95% confident that the true value p as estimated by $\hat{p}$ will fall into an interval $\pm$ 9.5% of $\hat{p}$, assuming the worst case, that is, if $\hat{p}$ were equal to 0.5. This $\hat{p}$ is the value of p that would be approached for an infinite number of simulation experiments (large LIMIT).

2.6 Verification and Validation

AIRDEF was thoroughly verified in two ways. First, for every small subset of program statements that constitutes a separate logical unit or operation, manual examination of the output as compared to the input was done on sets of input that represented each possible path through the unit. This was accomplished by inserting print statements at all key places during the debug phase of the programming, and by selected runs of the complete program with special combinations of input parameters. Second, a series of about 40 verification hypotheses was tested; each hypothesis was of the form "if the program is correct, a (specified change) in the input should cause a (specified change) in the output." AIRDEF was found to be robust -- incapable of yielding unreasonable answers -- under a wide range of reasonable and unreasonable parameters and combinations of parameters. Special attention was paid to conditions that would cause survivabilities to approach 0 or 1.

No objective real-world data are available for true validation of AIRDEF. According to established validation standards (7,14) the proper validation procedure would consist of running a series of simulation experiments with input parameters set equal to those for an actual series of battles whose outcomes are known; if the simulated survivabilities and other statistics output by AIRDEF were found to match closely the actual survivabilities, AIRDEF would be considered validated. Such primary validation would provide reasonable confidence in the results to be obtained in practice when AIRDEF will be used to predict survivabilities under conditions not yet actually encountered. A moderate number of actual battles of the kind modeled by AIRDEF has occurred in recent years, but the detailed data available on any one set of conditions are either of such small sample size or so highly classified that none could be released for the present study.

Secondary validation -- validation of AIRDEF against more elaborate simulations already validated -- is suggested by the existence of TACOS and other large-scale programs. However, it was found that none of the large-scale programs have received primary validation; in fact, none has been run a sufficiently large number of times to develop a low-variance estimate of survivability under any one set of conditions (4).

A much weaker form of validation has been accomplished by submitting the detailed results of the simulation study described in Chapter III to the Air Warfare Division of the United States Army Materiel Systems Analysis Agency. After comparing the results in detail with the experience of several qualified air defense analysts, Mr. John Meredith certified to the author on April 27, 1976, that AIRDEF produces results which very closely approximate the survivabilities of real-world air-to-ground combat situations.

CHAPTER III

DEMONSTRATION AND APPLICATION OF THE METHODOLOGY

3.1 Introduction

There are no intrinsic limits in AIRDEF on the number of combatants on either side, on the number of estimates generated or on the time increment used. Each of these factors influences the amount of computer resources used; under comparable conditions, AIRDEF uses orders of magnitude less computer resources than that required by TACOS and similar simulations. This chapter presents sample uses of AIRDEF for applications involving 4 air defense sites and 4 to 12 attacking aircraft, with a 12-second time increment. On a CDC 6600 computer, resource usages of up to 160.95 SRU were encountered in running 225 simulated battles (an SRU is an equivalent second of computer time, corrected to include usage of memory, input/output devices and other peripherals). The structure of the simulation, verified with spot checks, indicates that tactically reasonable runs involving up to 8 sites and 20 aircraft would use no more than 450 SRU for 225 battles with the 12-second time increment. If the time increment is divided by a given factor, the computer usage is increased by slightly less than the same factor; at present speeds and capabilities, no less than a 6-second time increment would be necessary for essentially maximal accuracy, so that usages requiring no more than 900 SRU for 8 sites and 20 aircraft are not expected, when 225 battles are simulated.

3.2 Programming AIRDEF

Prior to executing AIRDEF the user must determine certain aircraft and air defense parameters. This set-up phase of AIRDEF is meant to be as brief as possible, a distinct improvement over TACOS.

3.2.1 Aircraft Parameters

AIRDEF interactively questions the user during initialization to determine the number, position, speed, and engagement range of attacking aircraft. The only question which is not straightforward is aircraft positioning. AIRDEF deals only with the X-dimension in computing slant range and in moving aircraft, which may require a conversion of the standard X-Y-Z coordinate axis system. Consider Figure 9a below which depicts via the Universal Trans-Mercator grid system an aircraft at DH5065 attacking from northwest to southeast a target at DJ2520. For use in AIRDEF a conversion is made by superimposing a left-handed X-Y grid on the standard map, as seen in Figure 9b. From the AIRDEF conversion grid the attacking aircraft coordinates are read as (98,08), while the target is read as (10,15). The aircraft vs. air defense site configuration used in this chapter is shown in AIRDEF conversion representation in Figure 10.

Although uniformity of parameters is not required by AIRDEF, all attacking aircraft will be simulated for demonstration as flying at 950 knots, 15 kilometers altitude, and having an engagement range of 60 kilometers. These aircraft are equipped with radar homing anti-air defense missiles which have a uniform kill probability throughout their effective range of 0.90. Should the user desire to change the weapons carried or the kill probability distribution SUBROUTINE AKILL(SAM) must be modified

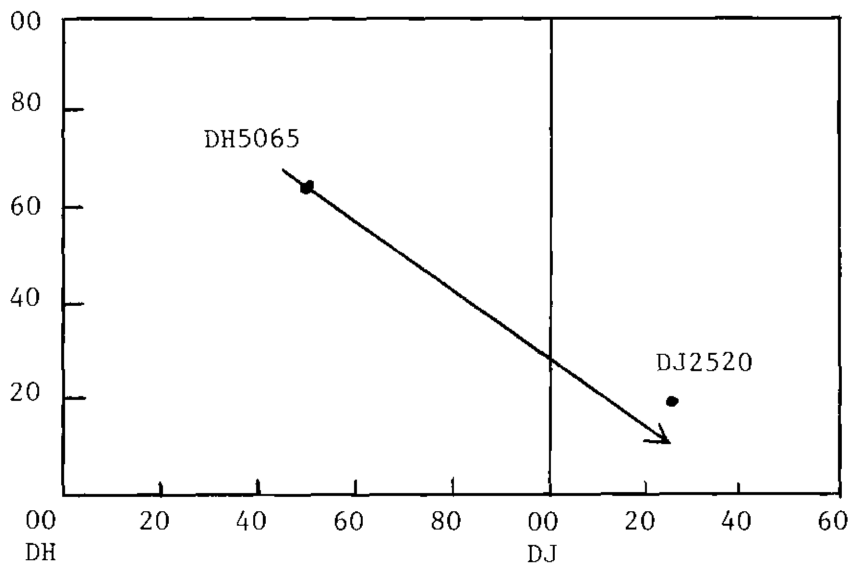

Figure 9a. Standard UTM Coordinate System

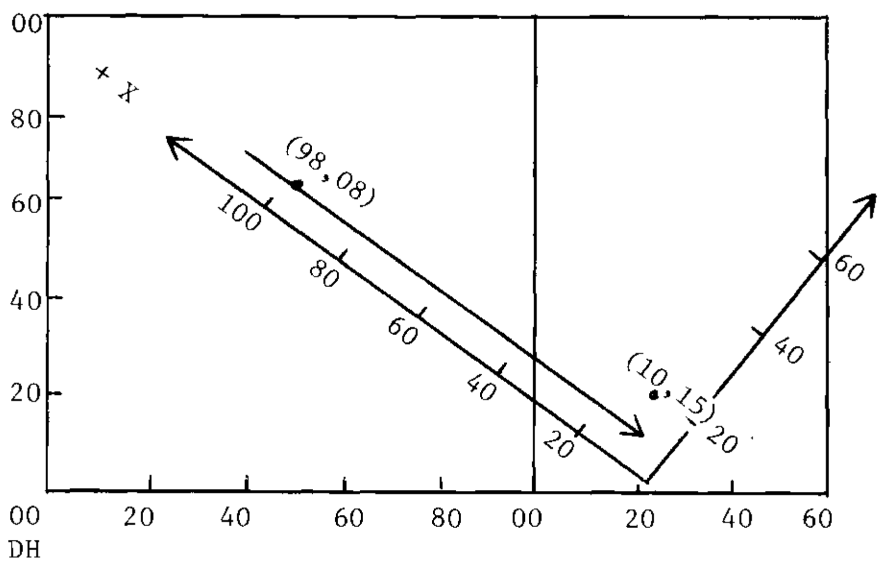

Figure 9b. AIRDEF Conversion Grid

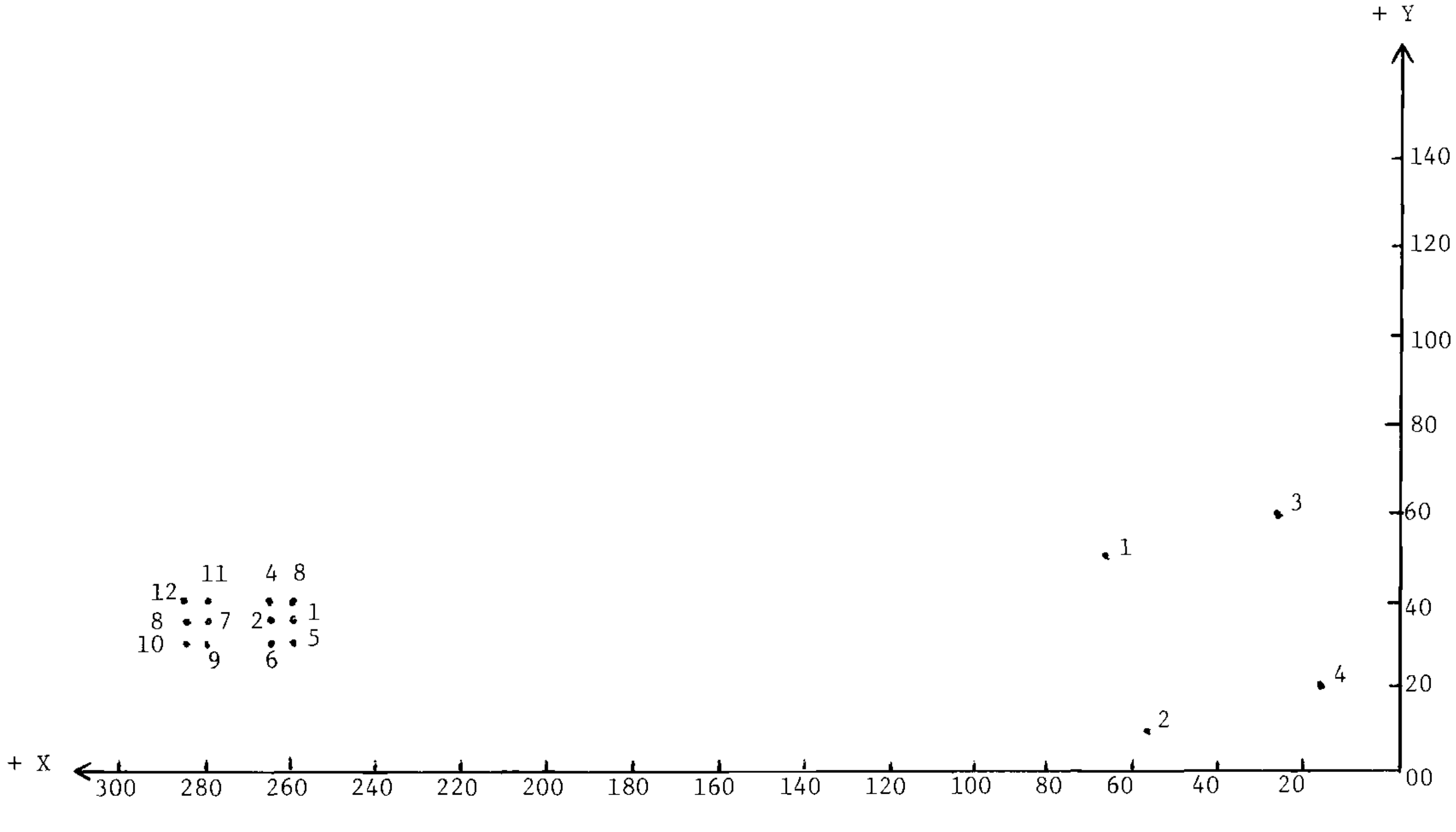

Figure 10. AIRDEF Aircraft vs. Air Defense Site Representation

directly. This portion of AIRDEF is short, to the point, and should present no problems to the user (see Appendix A, page 73).

3.1.2 Air Defense Parameters

The location of simulated air defense sites must be converted in the same manner as aircraft locations. As with the aircraft, AIRDEF does not require uniformity of site parameters; however, uniformity is assumed by AIRDEF in its present form. Users desiring to vary site parameters must dimension these parameters, modify the interactive questions to query each site, and index all modified parameters in the body of AIRDEF. Demonstration sites will have the following parameters:

1. Delay time between missile firings will be 0.5 minutes,
2. maximum radar acquisition range of 250 kilometers, acquisition probability of 0.95,
3. minimum tracking time prior to missile firing of 0.5 minutes, and
4. missile launch probability of 0.90, aircraft kill probability of 0.80.

3.3 AIRDEF Output

A typical time-step output from a verification form of AIRDEF with the above parameters might look as follows:

```
************************************************

   CURRENT TIME IS     .60

************************************************

FØR AC(1)........................

RANGE BETWEEN AC(1) AND AD(1) IS          218.07
DETECTIØN ØF  AC(1) BY  AD(1) IS   PØSSIBLE
RANGE BETWEEN AC(1) AND AD(2) IS          228.90
DETECTIØN ØF  AC(1) BY  AD(2) IS   PØSSIBLE
RANGE BETWEEN AC(1) AND AD(3) IS          258.69
RANGE TØØ GREAT FØR DETECTIØN BY AD(3)
RANGE BETWEEN AC(1) AND AD(4) IS          267.88
RANGE TØØ GREAT FØR DETECTIØN BY AD(4)
AC(1) HAS NØT BEEN TRACKED LØNG ENØUGH BY AD(1)
AC(2) HAS TRACKED LØNG ENØUGH AND IS READY TØ FIRE
RTF(1,1)= 0.; F(1,1)= 0.; FRNG(1)= 99999.00
RTF(1,2)= 2.; F(1,2)= 0.; FRNG(2)=   228.90
RTF(1,3)= 0.; F(1,3)= 0.; FRNG(3)= 99999.00
RTF(1,4)= 0.; F(1,4)= 0.; FRNG(4)= 99999.00
AD(2) WILL ATTEMPT TØ FIRE
UN( 11)= .4888
AD(2) WILL INTERCEPT AT  3.57

FØR AC(2)........................

RANGE BETWEEN AC(2) AND AD(1) IS          217.79
DETECTIØN ØF  AC(2) BY  AD(1) IS   PØSSIBLE
RANGE BETWEEN AC(2) AND AD(2) IS          229.50
DETECTIØN ØF  AC(2) BY  AD(2) IS   PØSSIBLE
RANGE BETWEEN AC(2) AND AD(3) IS          258.25
RANGE TØØ GREAT FØR DETECTIØN BY AD(3)
RANGE BETWEEN AC(2) AND AD(4) IS          268.21
RANGE TØØ GREAT FØR DETECTIØN BY AD(4)
AC(2) HAS NØT BEEN TRACKED LØNG ENØUGH BY AD(1)
AC(2) HAS TRACKED LØNG ENØUGH AND IS READY TØ FIRE
RTF(2,1)= 0.; F(2,1)= 0.; FRNG(1)= 99999.00
RTF(2,2)= 2.; F(2,2)= 1.; FRNG(2)= 99999.00
RTF(2,3)= 0.; F(2,3)= 0.; FRNG(3)= 99999.00
RTF(2,4)= 0.; F(2,4)= 0.; FRNG(4)= 99999.00
NØ SITES ARE REACY TØ FIRE

FØR AC(3)........................
```

```
RANGE BETWEEN AC(3) AND AD(1) IS           223.05
DETECTIØN ØF  AC(3) BY  AD(1) IS   PØSSIBLE
RANGE BETWEEN AC(3) AND AD(2) IS           233.86
DETECTIØN ØF  AC(3) BY  AD(2) IS   PØSSIBLE
RANGE BETWEEN AC(3) AND AD(3) IS           263.66
RANGE TØØ GREAT FØR DETECTIØN BY AD(3)
RANGE BETWEEN AC(3) AND AD(4) IS           272.86
RANGE TØØ GREAT FØR DETECTIØN BY AD(4)
AC(1) HAS TRACKED LØNG ENØUGH AND IS READY TØ FIRE
AC(2) HAS TRACKED LØNG ENØUGH AND IS READY TØ FIRE
RTF(3,1)= 1.; F(3,1)= 0.; FRNG(1)=   223.05
RTF(3,2)= 2.; F(3,2)= 1.; FRNG(2)= 99999.00
RTF(3,3)= 0.; F(3,3)= 0.; FRNG(3)= 99999.00
RTF(3,4)= 0.; F(3,4)= 0.; FRNG(4)= 99999.00
AD(1) WILL ATTEMPT TØ FIRE
UN( 12)= .3600
AD(1) WILL INTERCEPT AT  3.49

FØR AC(4)...........................

RANGE BETWEEN AC(4) AND AD(1) IS           222.77
DETECTIØN ØF  AC(4) BY  AD(1) IS   PØSSIBLE
RANGE BETWEEN AC(4) AND AD(2) IS           234.45
DETECTIØN ØF  AC(4) BY  AD(2) IS   PØSSIBLE
RANGE BETWEEN AC(4) AND AD(3) IS           263.23
RANGE TØØ GREAT FØR DETECTIØN BY AD(3)
RANGE BETWEEN AC(4) AND AD(4) IS           273.19
RANGE TØØ GREAT FØR DETECTIØN BY AD(4)
AC(1) HAS TRACKED LØNG ENØUGH AND IS READY TØ FIRE
AC(2) HAS TRACKED LØNG ENØUGH AND IS READY TØ FIRE
RTF(4,1)= 1.; F(4,1)= 1.; FRNG(1)= 99999.00
RTF(4,2)= 2.; F(4,2)= 1.; FRNG(2)= 99999.00
RTF(4,3)= 0.; F(4,3)= 0.; FRNG(3)= 99999.00
RTF(4,4)= 0.; F(4,4)= 0.; FRNG(4)= 99999.00
NØ SITES ARE REACY TØ FIRE

**************************************************

   CURRENT TIME IS     .80
```

The output for time .60 was compiled from the output of a verification run and is intended to illustrate both the verification form of AIRDEF and the complete AIRDEF process discussed in Chapter II.

The standard AIRDEF simulation would look as follows:

```
READY.
RUN

 76/05/20. 10.22.04.
PRØGRAM   INTDEF

 ENTER THE NUMBER ØF ESTIMATES ØF ADSSP DESIRED (<101)
? 3
 ENTER ANY SIX-DIGIT SEED FØR THE RANDØM NUMBER GENERATØR
? 161719
 ENTER THE NUMBER ØF BATTLES DESIRED FØR EACH ESTIMATE
? 75
 ENTER THE NUMBER ØF (AIRCRAFT,AIR DEFENSE) DESIRED
? 4,4
 ENTER THE X,Y,Z CØØRDINATES FØR AIRCRAFT  1
? 300,35,15
 ENTER THE X,Y,Z CØØRDINATES FØR AIRCRAFT  2
? 300,40,15
 ENTER THE X,Y,Z CØØRDINATES FØR AIRCRAFT  3
? 305,35,15
 ENTER THE X,Y,Z CØØRDINATES FØR AIRCRAFT  4
? 305,40,15
 ENTER THE X,Y,Z CØØRDINATES FØR AIR DEFENSE SITE  1
? 70,50,0
 ENTER THE X,Y,Z CØØRDINATES FØR AIR DEFENSE SITE  2
? 60,10,0
 ENTER THE X,Y,Z CØØRDINATES FØR AIR DEFENSE SITE  3
? 30,60,0
 ENTER THE X,Y,Z CØØRDINATES FØR AIR DEFENSE SITE  4
? 20,20,0
 ENTER THE VELØCITY (IN KNØTS) ØF AIRCRAFT  1
? 700
 ENTER THE VELØCITY (IN KNØTS) ØF AIRCRAFT  2
? 700
 ENTER THE VELØCITY (IN KNØTS) ØF AIRCRAFT  3
? 700
 ENTER THE VELØCITY (IN KNØTS) ØF AIRCRAFT  4
? 700
 ENTER AIRCRAFT'S ENGAGEMENT RANGE (KM'S) ØF AIR DEFENSE
? 90
 ENTER DELAY TIME (IN % ØF MIN.) BETWEEN MISSILE FIRINGS
? .5
 ENTER  MAX ACQUISITIØN RANGE (IN KM'S) FØR AIR DEFENSE SITES
? 2590
 ENTER MIN TRACKING TIME (% ØF MIN.) REQUIRED BEFØRE AD LAUNCH
? .5
```

```
 ENTER (IN KNØTS) THE SPEED ØF THE AIR DEFENSE MISSILES
? 1800
 ENTER TIME ADVANCE DESIRED (% ØF MINUTE) FØR SIMULATIØN
? .2
 ENTER THE AIR DEFENSE ACQUISITIØN PRØBABILITY
? .6
 ENTER THE NUMBER ØF RADAR SWEEPS PER MINUTE DESIRED
? 5
 ENTER THE AIR DEFENSE LAUNCH PRØBABILITY
? .9
 ENTER THE AIR DEFENSE MISSILE KILL PRØBABILITY
? .8


SURVIVAL PRØBABILITY FØR AIR DEFENSE SITE (1) = .6400
SURVIVAL PRØBABILITY FØR AIR DEFENSE SITE (2) = .7200
SURVIVAL PRØBABILITY FØR AIR DEFENSE SITE (3) = .8667
SURVIVAL PRØBABILITY FØR AIR DEFENSE SITE (4) = .8667

                         EXPECTED SURVIVABILITY =  .7733

NUMBER TIMES 0 SITES SURVIVED = 10.0 ; % =  .1333
NUMBER TIMES 1 SITE  SURVIVED =  0.0 ; % = 0.0000
NUMBER TIMES 2 SITES SURVIVED = 11.0 ; % =  .1467
NUMBER TIMES 3 SITES SURVIVED =  6.0 ; % =  .0800
NUMBER TIMES 4 SITES SURVIVED = 48.0 ; % =  .6400

SURVIVAL PRØBABILITY FØR AIR DEFENSE SITE (1) = .5733
SURVIVAL PRØBABILITY FØR AIR DEFENSE SITE (2) = .6133
SURVIVAL PRØBABILITY FØR AIR DEFENSE SITE (3) = .7067
SURVIVAL PRØBABILITY FØR AIR DEFENSE SITE (4) = .7067

                         EXPECTED SURVIVABILITY =  .6500

NUMBER TIMES 0 SITES SURVIVED = 22.0 ; % =  .2933
NUMBER TIMES 1 SITE  SURVIVED =  0.0 ; % = 0.0000
NUMBER TIMES 2 SITES SURVIVED =  7.0 ; % =  .0933
NUMBER TIMES 3 SITES SURVIVED =  3.0 ; % =  .0400
NUMBER TIMES 4 SITES SURVIVED = 43.0 ; % =  .5733

SURVIVAL PRØBABILITY FØR AIR DEFENSE SITE (1) = .4800
SURVIVAL PRØBABILITY FØR AIR DEFENSE SITE (2) = .5333
SURVIVAL PRØBABILITY FØR AIR DEFENSE SITE (3) = .7733
SURVIVAL PRØBABILITY FØR AIR DEFENSE SITE (4) = .7867

                         EXPECTED SURVIVABILITY =  .6433
```

```
NUMBER TIMES 0 SITES SURVIVED = 16.0 ; % =  .2133
NUMBER TIMES 1 SITE  SURVIVED =  1.0 ; % =  .0133
NUMBER TIMES 2 SITES SURVIVED = 18.0 ; % =  .2400
NUMBER TIMES 3 SITES SURVIVED =  4.0 ; % =  .0533
NUMBER TIMES 4 SITES SURVIVED = 36.0 ; % =  .4800

   MEAN FØR AD(1)= .5644  VARIANCE= .0065
   MEAN FØR AD(2)= .6222  VARIANCE= .0088
   MEAN FØR AD(3)= .7822  VARIANCE= .0065
   MEAN FØR AD(4)= .7867  VARIANCE= .0064
  ØVERALL ADSSP ESTIMATE IS .6888888888889

SRU      37.495 UNTS.

RUN CØMPLETE.
```

As can be seen, three estimates of ADSSP each requiring 75 battles of four aircraft vs. four air defense sites were generated. This is the form of AIRDEF most users would be likely to use.

3.4 An AIRDEF Application

As straightforward as AIRDEF is in its preparation and execution, large battles may take longer than the user desires to spend. Additionally, computer facilities may not always be instantly available every time ADSSP estimates are needed. For these reasons a more managable method of generating ADSSP estimates for a narrow range of aircraft parameters is desirable. Providing that a large number of AIRDEF estimates of ADSSP are available covering the narrow range of aircraft parameters of interest, regression analysis is appropriate to derive a predictive equation which will then be valid throughout this narrow range. Such an equation can be used by tactical commanders at all levels to estimate ADSSP when the full simulation cannot be executed.

3.4.1 The Model

Given a particular type air defense missile system, the problem of estimating the survivability of missile sites will depend in large on the type of attack anticipated. The aircraft factors considered here are number, speed, air defense engagement range, and electronic countermeasures (ECM) employed. This is stated symbolically as

$Y = f(\alpha + \beta + \gamma + \delta)$, where

Y = estimate of survivability

α = number of aircraft in the attack

β = aircraft attack speed

γ = aircrafts' engagement range

δ = ECM capability of aircraft.

A 3^4 fixed effects experimental design model was used with three levels of the four aircraft effects seen in Table 2.

Table 2. Factor Levels of Aircraft for 3^4 Aircraft Problem

	Level		
Effect	1	2	3
A' = # aircraft	4	8	12
B' = AC Speed, Knots	450	700	950
C' = Attack Range, Km.	30	60	90
D' = ECM Capability	.95	.60	.25

AIRDEF was utilized to generate three estimates, each based upon 75 battles, for all 81 factor combinations. The entries of Table 3a, Table 3b, and Table 3c are the results of the AIRDEF executions. The parenthetical entries are the transformations discussed by Bartlett (1), that is,

$$Y = \text{Arcsin}\ (\text{estimate}^2) \quad .$$

This transformation is necessary to insure homogeniety of variance between cells, which is required for ANOVA. Prior to using ANOVA and regression computer routines the data representing the four aircraft effects was coded using the following relations:

$$A = \frac{(\text{Number of Attacking Aircraft})}{4} \ ,$$

$$B = 1 + .004\ (\text{Aircraft Attack Speed} - 450),$$

$$C = \frac{(\text{Aircraft Attack Range})}{30} \ ,$$

$$D = 1 + 2.857\ (.95 - \text{ECM\%})$$

This coding allows the values of A', B', C', and D' to correspond to the levels of Table 2, above.

The data of Tables 3a, 3b, and 3c were analyzed using a standard ANOVA computer program. Table 4 summarizes the results of the ANOVA program. The only effect not significant at α=0.05 was the BD interaction

Table 3a. ADSSP Estimates for Four Attacking Aircraft

Aircraft Attack Range (Kilometers)	ECM	Aircraft Attack Velocity (Knots) 450	700	950
		1.00 (1.5708)	.99 (1.5130)	.98 (1.4289)
	.95	1.00 (1.5708)	.99 (1.5130)	.99 (1.5130)
		1.00 (1.5708)	.99 (1.5130)	.96 (1.3872)
		.99 (1.5130)	.99 (1.4891)	.96 (1.3871)
30	.60	1.00 (1.5708)	1.00 (1.5708)	.97 (1.4068)
		1.00 (1.5708)	.99 (1.4891)	.97 (1.4068)
		1.00 (1.5708)	.99 (1.4891)	.95 (1.3453)
	.25	1.00 (1.5708)	.99 (1.4891)	.95 (1.3611)
		1.00 (1.5708)	.99 (1.4706)	.96 (1.3781)
		1.00 (1.5708)	.94 (1.3233)	.80 (1.1113)
	.90	.99 (1.4891)	.94 (1.3233)	.84 (1.1638)
		1.00 (1.4891)	.95 (1.3611)	.84 (1.1593)
		1.00 (1.5708)	.91 (1.2780)	.85 (1.1825)
60	.60	.99 (1.5130)	.94 (1.3233)	.81 (1.1198)
		1.00 (1.5708)	.97 (1.3967)	.77 (1.0786)
		.99 (1.4891)	.94 (1.3233)	.61 (.8963)
	.25	1.00 (1.5708)	.96 (1.3872)	.72 (1.0169)
		1.00 (1.5708)	.93 (1.3030)	.69 (.9803)
		.95 (1.3611)	.75 (1.0472)	.50 (.7921)
	.95	.93 (1.3164)	.70 (.9985)	.49 (.7754)
		.95 (1.3611)	.64 (.9343)	.59 (.8827)
		.96 (1.3781)	.78 (1.0866)	.48 (.7687)
90	.60	.93 (1.3164)	.68 (.9767)	.54 (.8288)
		.96 (1.3872)	.77 (1.0786)	.53 (.8188)

Table 3a. ADSSP Estimates for Four Attacking Aircraft (Continued)

Aircraft Attack Range (Kilometers)	ECM	Aircraft Attack Velocity (Knots) 450	700	950
		.93 (1.3096)	.69 (.9839)	.41 (.6983)
	.25	.94 (1.3304)	.71 (1.0095)	.42 (.7118)
		.95 (1.3531)	.64 (.9273)	.50 (.7854)

Table 3b. ADSSP Estimates for Eight Attacking Aircraft

Aircraft Attack Range (Kilometers)	ECM	Aircraft Attack Velocity (Knots) 450	700	950
30	.95	1.00 (1.5708)	.97 (1.3967)	.87 (1.2069)
		1.00 (1.5708)	.98 (1.4289)	.83 (1.1503)
		1.00 (1.5708)	.95 (1.3611)	.85 (1.1731)
	.60	1.00 (1.5708)	.96 (1.3781)	.83 (1.1458)
		1.00 (1.5708)	.96 (1.3694)	.79 (1.0948)
		1.00 (1.5708)	.97 (1.3967)	.81 (1.1283)
	.25	1.00 (1.5708)	.94 (1.3233)	.67 (.9624)
		1.00 (1.5708)	.93 (1.3096)	.68 (.9731)
		1.00 (1.5708)	.93 (1.3030)	.59 (.8759)
60	.95	.97 (1.4068)	.61 (.8963)	.13 (.3787)
		.95 (1.3611)	.63 (.9238)	.11 (.3486)
		.95 (1.3611)	.64 (.9273)	.15 (.4070)
	.60	.92 (1.2902)	.56 (.8455)	.11 (.3381)
		.95 (1.3531)	.67 (.9624)	.10 (.3273)
		.98 (1.4413)	.43 (.7219)	.13 (.3689)
	.25	.94 (1.3233)	.40 (.6915)	.06 (.2612)
		.94 (1.3233)	.43 (.7185)	.07 (.2806)
		.95 (1.3611)	.36 (.6435)	.03 (.1741)
90	.95	.73 (1.0244)	.07 (.2742)	.00
		.69 (.9875)	.15 (.3977)	.00
		.74 (1.0434)	.10 (.3327)	.00
	.60	.73 (1.0319)	.09 (.3047)	.00
		.73 (1.0244)	.07 (.2742)	.00
		.73 (1.0282)	.08 (.2868)	.00
	.25	.67 (.9660)	.06 (.2612)	.00
		.63 (.9283)	.03 (.1741)	.00
		.61 (.8963)	.06 (.2475)	.00

Table 3c. ADSSP Estimates for 12 Attacking Aircraft

Aircraft Attack Range (Kilometers)	ECM	Aircraft Attack Velocity (Knots) 450	700	950
		1.00 (1.5708)	.75 (1.0511)	.32 (.6013)
	.95	.99 (1.4706)	.81 (1.1283)	.35 (.6365)
		.99 (1.5130)	.79 (1.0989)	.35 (.6365)
		.99 (1.4891)	.70 (.9948)	.21 (.4801)
30	.60	1.00 (1.5708)	.72 (1.0206)	.25 (.5313)
		.99 (1.4706)	.71 (1.0021)	.24 (.5120)
		.98 (1.4413)	.54 (.8288)	.07 (.2742)
	.25	.98 (1.4551)	.53 (.8154)	.05 (.2555)
		.99 (1.4891)	.51 (.7954)	.06 (.2475)
		.78 (1.0826)	.03 (.1836)	.00
	.95	.81 (1.1198)	.02 (.1419)	.00
		.78 (1.4891)	.05 (.2330)	.00
		.70 (.9948)	.01 (.1295)	.00
60	.60	.77 (1.0746)	.01 (.8817)	.00
		.79 (1.1030)	.02 (.1419)	.00
		.57 (.8556)	.00	.00
	.25	.57 (.8556)	.00	.00
		.62 (.9100)	.01 (.0817)	.00
		.19 (.4595)	.00	.00
		.15 (.4023)	.00	.00
		.12 (.3639)	.00	.00
		.12 (.3639)	.00	.00
90	.60	.13 (.3738)	.00	.00
		.15 (.3977)	.00	.00
		.06 (.2475)	.00	.00
	.25	.01 (.1295)	.00	.00
		.04 (.2097)	.00	.00

Table 4. ANOVA for 3^4 Aircraft Problem

Source	df	SS	MS	F
A, # of AC	2	25.782	12.891	9423.24
B, Speed	2	18.8416	9.4208	6886.55
C. Range	2	22.4354	11.2177	8200.07
D, ECM	2	.4431	.2215	161.915
AB	4	1.9456	.4864	355.55
AC	4	2.2208	.5552	405.85
AD	4	.0629	.0157	11.47
BC	4	.8291	.2071	151.46
BD	4	.0154	.0038	2.814
CD	4	.0351	.0087	6.4108
ABC	8	2.517	.3146	229.9707
ABD	8	.049	.00612	4.4736
ACD	8	.052	.0065	4.7514
BCD	8	.1178	.0147	10.76
ABCD	16	.1378	.0085	6.27
Error + Reps	162	.2216	.001368	

$F_{.05,2,162} = 3.05$

term.

Using the results of Table 4 a linear regression model was postulated from the more significant effects found in the ANOVA. The complete model was:

$$Y = \beta_0 + \beta_1 A^3 + \beta_2 B^3 + \beta_3 C^3 + \beta_4 D^3 + \beta_5 A^2 + \beta_6 B^2 + \beta_7 C^2 + \beta_8 D^2$$

$$+ \beta_9 A + \beta_{10} B + \beta_{11} C + \beta_{12} D + \beta_{13} AB + \beta_{14} AC + \beta_{15} BC + \beta_{16} ABC$$

$$+ \beta_{17} BCD.$$

The results of the step-wise regression program are summarized in Table 5. It should be pointed out that the above regression equation was derived using only the ADSSP estimates which were less than .90 and greater than .20. This was done to produce a more accurate model; it was found that attempting to predict ADSSP when the entire data set was used could not be satisfactorily accomplished. The final regression prediction equation is

$$Y = .73333 - .202692(ABC) - .010332(D^3) + .0112664(BCD)$$

$$- .177563(A^2) + .930932(A) + .109303(BC) - .0547531(C^2) \qquad (3)$$

$$+ .184061(C).$$

Table 5. Step-wise Regression of 3^4 Aircraft Problem

```
STEP NØ. 8
THE LAST VAR. ENTERED 8
SUM ØF SQUARES REDUCED 3.46974E-2
PRØPØRTIØN REDUCED 1.52849E-2
CUMULATIVE SS REDUCED 2.09488
CUMULATIVE PRØPØRTIØN REDUCED .922834  ØF 2.27005
NUMBER ØF VARIABLES ENTERED..... 8
MULTIPLE CØRRELATIØN CØEFFICIENT .960642
ADJ. MULTIPLE CØRRELATIØN CØEFF. .956618
F-VALUE FØR ANØVA(FØR THE REG.)  103.147
STANDARD ERRØR ØF ESTIMATE...... 5.03855E-2
ADJUSTED STD. ERRØR ØF ESTIMATE  5.28448E-2
```

VARIABLE NUMBER	REGRESSIØN CØEFFICIENT	STD.ERRØR ØF REG.CØEFF.	CØMPUTED T-VALUE
13 = ABC	-.202692	1.17807E-2	-17.2055
1 = D^3	-.010332	1.04441E-3	-9.89263
14 = BCD	1.12664E-2	2.84545E-3	3.95943
2 = A^2	-.177563	2.22979E-2	-7.96325
6 = A	.930932	.120636	7.71685
12 = BC	.109305	1.68148E-2	6.50053
4 = C^2	-5.47531E-2	1.23197E-2	-4.44435
8 = C	.184061	4.97872E-2	3.69694

INTERCEPT .733333

In developing Equation (3), it was decided not to use transcendental functions because of a traditional prejudice of tactical commanders against such functions, and because their digital computation is equivalent to evaluating many polynomial terms. Nine terms were needed to provide satisfactory goodness of fit; no attempt to limit this to fewer terms was made, since the tactical use of the equation will undoubtedly involve a preprogrammed microcomputer.

Equation (3) above is easily computerized and its execution time is rarely greater than 0.385 SRU on a CDC Cyber 70 computer. Appendix B contains an interactive program which makes use of the above regression equation to predict ADSSP based upon the user supplied aircraft data. Also found as output of that program is an approximate confidence interval on the true mean value of the response, air defense site survivability, for the specific set of aircraft conditions used. The development of the confidence interval equation is given in Draper and Smith; the equation is

$$CI = \hat{Y} \pm t_{(n-p-1,\ 1-\alpha)} \cdot s \cdot \sqrt{X_0{}'CX_0} \tag{4}$$

$\hat{Y}$ = regression prediction for a specific set of independent variables,

t = percentage point of the t-distribution,

n = number of data points used to determine the regression equation,

p = number of terms in the regression equation,

s = standard error of estimate, from the regression program,

X_0 = a column vector of specific independent variables, augumented by inserting 1 before the individual values,

X'_0 = the transpose of the X_0 vector,

X = the matrix formed by augumenting the matrix of independent variables used to find the regression equation with a column of 1's in the first column,

X' = the transpose of the X matrix

$C = (X'X)^{-1}$

This confidence interval reflects the accuracy of the regression equation predictions, and can be interpreted as meaning that there is a $(1-\alpha)$ probability that the true mean value of air defense site survivability at a specific set of aircraft attack conditions will fall in the confidence interval.

The confidence interval of Appendix B is a "rough" interval because equation (4) is not used to determine the interval. Equation (4) is used in the stepwise regression program found in Appendix C. Examination of the confidence intervals shown in Appendix C computer for 26 specific sets of aircraft parameters indicates that the average width of these 26 intervals is 12.30%, the maximum width is 16.64%, and the minimum interval is 8.84%, and only two times is the interval wider than 13.4%; therefore the program of Appendix B simple adds and subtracts 6.77% to the predicted value of ADSSP given by equation (3) and uses these as the upper and lower limits of a 95% confidence interval for specific sets of aircraft parameters used in the program of Appendix B.

3.4.2 An Example Problem

Equation (3) above can easily be solved by pocket calculators. Consider an attack by five aircraft flying at 600 knots air speed, attacking from a range of 45 kilometers, and having moderate ECM capability, say 0.6. This data must be transformed before substitution into equation (3) as follows:

A = (Number of Attacking Aircraft)/4

B = 1 + .004 (Aircraft Speed - 450)

C = (Aircraft Attack Range)/30

$$D = 1 + 2.857\ (.95 - ECM\%)$$

For the attack situation described above,

$$A = (5)/4 = 1.25$$

$$B = 1 + .004(600 - 450) = 1.6$$

$$C = (45)/30 = 1.5$$

$$D = 1 + 2.857(.95 - .6) = 2.0$$

Substitution into Equation (3) yields

$$Y = .73333 - .202692(1.25)(1.6)(1.5) - .010332(2^3)$$

$$+ .0112664(1.6)(1.5)(2) - .177563(1.25^2) + .930932(1.25)$$

$$+ .109303(1.6)(1.5) - .054731(1.5^2) + .184061(1.5) = 1.27289$$

This value must be transformed back using the reverse of Bartlett's transformation,

$$Y = (\sin(Y))^{.5} \tag{5}$$

Substitution of Y = 1.2728 into Equation (5) yields an estimate of 0.9777, a very high probability of survival.

Appendix C contains the results of the regression prediction equation when the original regression data generated by AIRDEF is used. The 99% confidence interval shown is for the true mean value of the response, air defense survivability. The computation of these intervals is done by using equation (4) and considering each specific set of aircraft parameters as the X_0 vector and augumenting that vector with a 1 in the first position as discussed previously.

Figure 11 shows both the regression predictions from equations (3) and (5) and AIRDEF (indicated by "X") estimates from AIRDEF simulations based upon 225 battles of how ADSSP varies as the number of attacking aircraft increases and as the performance of these attacking aircraft vary from low performance (speed = 450 knots, attack range = 30 km., ECM = .95) through medium performance (speed = 700 knots, attack range = 60 km., ECM = .6) to high performance (speed = 950 knots, attack range = 90 km., ECM = .25).

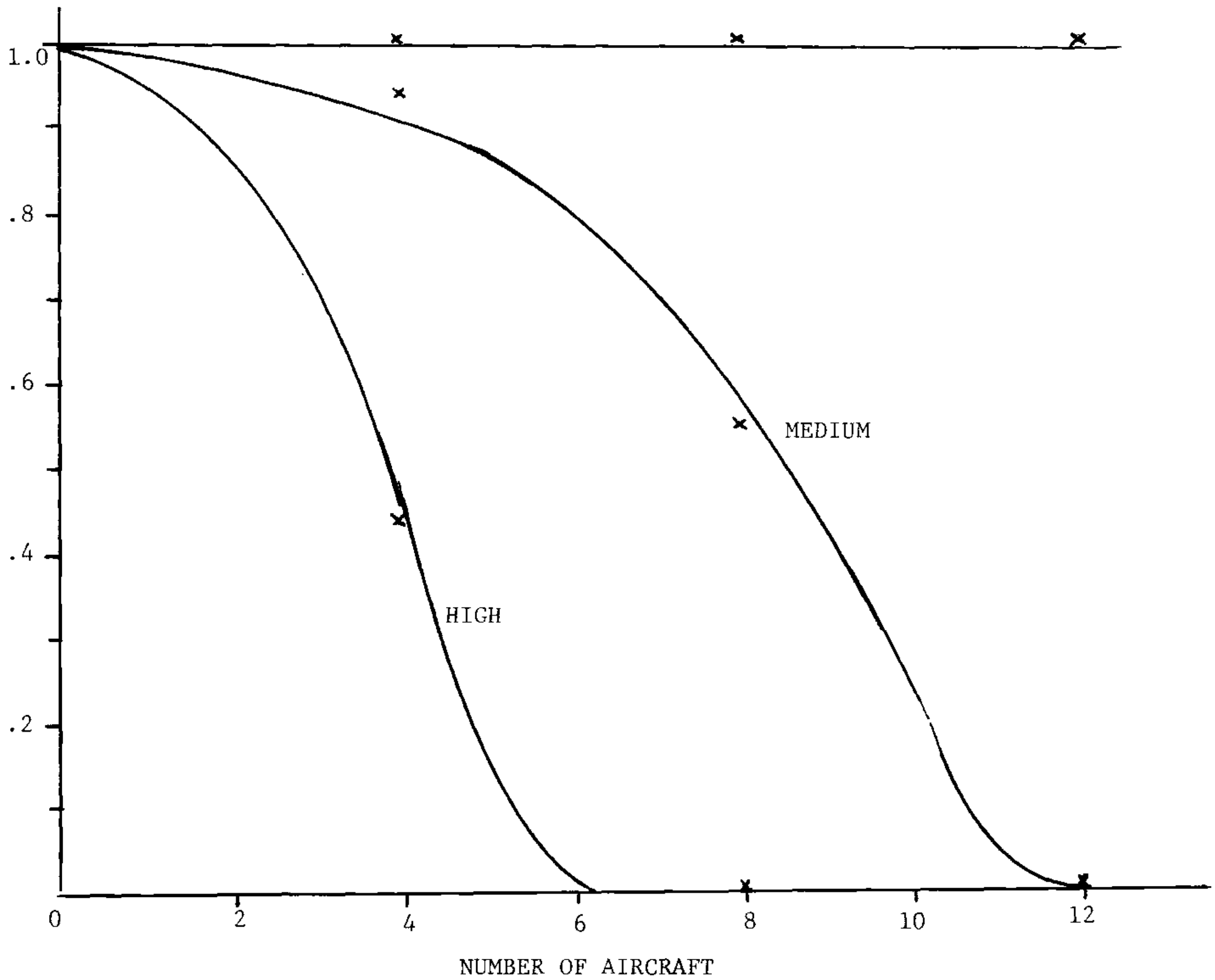

Figure 11. Survivability as Number of Aircraft and Performance of these Aircraft Vary.

CHAPTER IV

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

The application of time-step simulation for the estimation of air defense site survivability probability is a viable methodology. The AIRDEF simulation program presented, incorporating both high detail and low set-up and run times, is capable of generating virtually an unlimited number of estimates of ADSSP to the degree of accuracy required by the user in a very acceptable time.

The validity of AIRDEF output is established only by the review and approval of the Air Warfare Division, United States Army Materiel Systems Analysis Agency. This form of validation was necessary due to the lack of access to the very limited real world data available.

The use of step-wise regression analysis is appropriate to encapsulate the results of many AIRDEF estimates covering a narrow range of aircraft and air defense parameters. The regression equation obtained from this analysis may then be used in lieu of AIRDEF to generate ADSSP estimates which will closely approximate AIRDEF estimates. Care must be taken to insure that AIRDEF estimates for all combinations of the narrow range of parameters of interest are included in the regression analysis. Such regression equations would then allow tactical commanders to assess the impact of varying such factors as number of air defense sites deployed, the spatial arrangement of these sites, and the allocation of missiles to each site.

The time-step approach to simulating continuously changing systems such as air-to-ground battles is an appropriate technique. The savings in set-up and execution times greatly outweigh any loss of detail encountered.

5.2 Recommendations

The AIRDEF methodology as presented is a valid simulation of a high altitude air defense system; however, there are several areas of the methodology which could be improved upon. Many of these improvements are trivial in nature and could be easily accomplished by slight programming modifications. A partial list of these easily handled extensions includes 1) extending the number of combatants on both sides, 2) placing constraints on the number of missiles fired by each side, both in total and at each time step, 3) forcing the aircraft to use the same logic as the sites regarding site acquisition, tracking, and missile launching, 4) computing statistics on number of missiles fired, time to acquisition, missile time of flight, aircraft survivability, and range at kill.

Two non-trivial areas of interest are obvious, the need to vary the aircraft flight paths and the need for a simulation which will incorporate low altitude aircraft. The varying flight path problem could perhaps be handled with a subroutine which would compare the aircraft locations with predetermined, user-supplied "check points." After the aircraft have passed each check point the subroutine would change the flight path of the aircraft.

Research into the applicability of using the AIRDEF approach to modeling a low-altitude missile system should be undertaken. The major problem here will be to find a means of introducing the affects of terrain and the "terrain masking" problem. One of the primary advantages of AIRDEF is that it does not generate a terrain tape for each site vs. aircraft. If this time savings is to be maintained some easily inserted subroutine must be developed; perhaps an aircraft altitude check based upon the X-Y range would be sufficient.

In addition, the time of intercept should be determined for a "tail shot." This situation is likely to be encountered and can easily be handled by keeping a flag to indicate decreasing-increasing ranges between aircraft and sites. When the range is increasing a tail shot must be made. In that case the velocity of the aircraft must be subtracted from the velocity of the missile prior to dividing into the slant range between the two combatants.

A P P E N D I C E S

APPENDIX A

This appendix contains a complete FORTRAN listing of the AIRDEF simulation program. The program is interactive; inputs are made in free-field format. Insertion of the appropriate "CALL PRINT (??)" statement where indicated by commend cards will result in a verification printing as found in Chapter III.

```
PROGRAM AIRDEF(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
COMMON/1/AC(12),AD(4),INT(12),D(12,4),R(12,4),H(12,4)
COMMON/2/TT(12,4),FRNG(4),RTF(12,4),TI(12,4),F(12,4),
*  A(12)
COMMON/3/B(12),C(12),X(4),Y(4),Z(4),ACVOL(12),UN(500)
COMMON/4/MAXRNG(12,4),J,K,M,NI,SUMJ,SUMK,KT,IT
COMMON/5/ACKILL
COMMON/7/DEF0,DEF1,DEF2,DEF3,DEF4,LIMIT,PC0,PC1,PC2,
*  PC3,PC4
DIMENSION SUMAC(12),SUMAD(12),AVEAC(12),AVEAD(12)
DIMENSION AAD(100,4),SUMAAD(4),SUMSQR(4),ADMEAN(4),
*  VAR(4)
DIMENSION A1(12),B1(12),C1(12)
DIMENSION X1(4),Y1(4),Z1(4)
DIMENSION ACVOL1(12)
REAL IT
C*****************************************************
C  THE FOLLOWING READ AND PRINT STATEMENTS COMPRISE
C  THE INTERACTIVE PORTION OF THE SIMULATION.
C*****************************************************
PRINT(6,*)"ENTER THE NUMBER OF ESTIMATES OF ADSSP
*  DESIRED (<101)"
READ(5,*) IN
PRINT(6,*)"ENTER ANY SIX-DIGIT SEED FOR THE RANDOM
*  NUMBER GENERATO
READ(5,*) SEED
DO 250 IA=1,4
DO 251 IB=1,100
251 AAD(IB,IA)=0
SUMAAD(IA)=0
SUMSQR(IA)=0
ADMEAN(IA)=0
250 VAR(IA)=0
PRINT(6,*)"ENTER THE NUMBER OF BATTLES DESIRED FOR
*  EACH ESTIMATE"
READ(5,*) LIMIT
PRINT(6,*)"ENTER THE NUMBER OF (AIRCRAFT,AIR DEFENSE)
*  DESIRED"
READ(5,*)MA,NAD
DO 300 I=1,MA
PRINT(6,*)"ENTER THE X,Y,Z COORDINATES FOR AIRCRAFT
*  ",I
300 READ(5,*)A1(I),B1(I),C1(I)
DO 301 J=1,NAD
PRINT(6,*)"ENTER THE X,Y,Z COORDINATES FOR AIR
*  DEFENSE SITE  ",J
301 READ(5,*) X1(J),Y1(J),Z1(J)
DO 302 K=1,MA
PRINT(6,*)"ENTER THE VELOCITY (IN KNOTS) OF AIRCRAFT
*  ",K
```

```
302 READ(5,*)ACVOL1(K)
PRINT(6,*)"ENTER AIRCRAFT"S ENGAGEMENT RANGE (KM"S)
    *  OF AIR DEFENSE
READ(5,*)DCRIT1
PRINT(6,*)"ENTER DELAY TIME (IN % OF MIN.) BETWEEN
    *  MISSILE FIRINGS
READ(5,*)RECT1
PRINT(6,*)"ENTER  MAX ACQUISITION RANGE (IN KM"S) FOR
    *  AIR DEFENSE
READ(5,*)MAXRNG1
PRINT(6,*)"ENTER MIN TRACKING TIME (% OF MIN.)
    *  REQUIRED BEFORE AD
READ(5,*)CTT1
PRINT(6,*)"ENTER (IN KNOTS) THE SPEED OF THE AIR
    *  DEFENSE MISSILES"
READ(5,*) ADMSLS1
PRINT(6,*)"ENTER TIME ADVANCE DESIRED (% OF MINUTE)
    *  FOR SIMULATION
READ(5,*)TIMING1
PRINT(6,*)"ENTER THE AIR DEFENSE ACQUISITION
    *  PROBABILITY"
READ(5,*)ACQUIR1
PRINT(6,*)"ENTER THE NUMBER OF RADAR SWEEPS PER
    *  MINUTE DESIRED"
READ(5,*)RSPM
ACQUIR2=1-(1-ACQUIR1)**(TIMING1*RSPM)
PRINT(6,*)"ENTER THE AIR DEFENSE LAUNCH PROBABILITY"
READ(5,*) GOOD1
PRINT(6,*)"ENTER THE AIR DEFENSE MISSILE KILL
    *  PROBABILITY"
READ(5,*) ADKILL1
C***********************************************************
C  THE  "DO 200" LOOP CONTROLS THE NUMBER OF ADSSP
C  ESTIMATES THE SIMULATION WILL PRODUCE.
C***********************************************************
DO 200 LOOPS=1,IN
DO 100 KEEP=1,12
SUMAC(KEEP)=0
SUMAD(KEEP)=0
AVEAC(KEEP)=0
100 AVEAD(KEEP)=0
DEF0=0
DEF1=0
DEF2=0
DEF3=0
DEF4=0
C***********************************************************
C  THE "DO 101" LOOP CONTROLS THE NUMBER OF BATTLES
C  RUN FOR EACH ESTIMATE.
C***********************************************************
```

```
DO 101 LOOP=1,LIMIT
M=MA
N=NAD
A(1)=A1(1)
B(1)=B1(1)
A(2)=A1(2)
B(2)=B1(2)
A(3)=A1(3)
B(3)=B1(3)
A(4)=A1(4)
B(4)=B1(4)
A(5)=A1(5)
B(5)=B1(5)
A(6)=A1(6)
B(6)=B1(6)
A(7)=A1(7)
B(7)=B1(7)
A(8)=A1(8)
B(8)=B1(8)
A(9)=A1(9)
B(9)=B1(9)
A(10)=A1(10)
B(10)=B1(10)
A(11)=A1(11)
B(11)=B1(11)
A(12)=A1(12)
B(12)=B1(12)
DO 1 J=1,M
C(J)=C1(J)
ACVOL(J)=ACVOL1(J)
ACVOL(J)=ACVOL(J)*.03086
INT(J)=0
AC(J)=1
1 CONTINUE
X(1)=X1(1)
Y(1)=Y1(1)
Z(1)=Z1(1)
X(2)=X1(2)
Y(2)=Y1(2)
Z(2)=Z1(2)
X(3)=X1(3)
Y(3)=Y1(3)
Z(3)=Z1(3)
X(4)=X1(4)
Y(4)=Y1(4)
Z(4)=Z1(4)
DCRIT=DCRIT1
RECT=RECT1
DO 2 J=1,M
DO 3 K=1,N
```

```
AD(K)=1
FRNG(K)=99999
H(J,K)=0
MAXRNG(J,K)=MAXRNG1
TT(J,K)=0
F(J,K)=0
RTF(J,K)=0
C+++++++++++++++++++++++++++++++++++++++++++++
C  THE Y-Z RANGE COMPONENT IS COMPUTED HERE.
C+++++++++++++++++++++++++++++++++++++++++++++
D(J,K)=(B(J)-Y(K))**2 + (C(J)-Z(K))**2
3 CONTINUE
2 CONTINUE
CTT=CTT1
KT=0
TIMLIM=20
IT=0
TIMING=TIMING1
ADMSLSP=ADMSLS1
ADMSLSP=ADMSLSP*.03086
ACQUIR=ACQUIR2
GOOD=GOOD1
ADKILL=ADKILL1
SEED=SEED+1
CALL RANSET(SEED)
DO 21 I=1,500
UN(I)=RANF(I)
21 CONTINUE
C*****************************************************
C  THIS IS THE STARTING POINT FOR EACH TIME INTERVAL.
C  CHECK TO SEE IF THE TIME LIMIT HAS EXPIRED.
C*****************************************************
20 IF(IT.GT.TIMLIM) GO TO 999
CONTINUE
C****************************************************
C* THE ABOVE "CONTINUE", REPLACED BY "CALL PRINT(25)"*
C* WOULD RESULT IN A VERIFICATION OUTPUT.            *
C*                                                   *
C* THE "DO 4" LOOP CONTROLS THE AIRCRAFT LOOKING AT  *
C* THE AIR DEFENSE SITES FOR A KILL DETERMINATION.   *
C*                                                   *
C* THE "CONTINUE" FOLLOWING "DO 4", IF REPLACED BY   *
C* "CALL PRINT(7)", WILL RESULT IN A VERIFICATION.   *
C****************************************************
DO 4 J=1,M
CONTINUE
C++++++++++++++++++++++++++++++++++++++++++++++++
C+ INITIALIZE RANGE INDICATOR BY MAKING SUMH = 0+
C++++++++++++++++++++++++++++++++++++++++++++++++
SUMH=0
```

```
C++++++++++++++++++++++++++++++++++++++++++++++++++++
C+ CHECK TO INSURE THAT AC(J) IS IN THE BATTLE AREA +
C++++++++++++++++++++++++++++++++++++++++++++++++++++
 IF((AC(J).EQ.0)) GO TO 40
 IF((IT.LT.INT(J))) GO TO 41
C*****************************************************
C*  THE "DO 5" LOOP CONTROLS THE AIR DEFENSE
C*  SITES AS EACH AIRCRAFT CHECKS FOR KILL
C*****************************************************
 DO 5 K=1,N
 IF(AD(K).EQ.0) GO TO 50
C++++++++++++++++++++++++++++
C+ SLANT RANGE DETERMINATION +
C++++++++++++++++++++++++++++
 R(J,K)=SQRT((A(J)-X(K))**2+D(J,K))
 CONTINUE
C++++++++++++++++++++++++++++++++++++++++++++++++++
C+  THE ABOVE "CONTINUE", WHEN REPLACED BY "CALL  +
C+  PRINT(7)", WILL RESULT IN A VERIFICATION      +
C+  PRINT OUT.                                    +
C+                                                +
C+  IF AC(J) IS CLOSE ENOUGH TO AD(K), ATTEMPT TO +
C+  KILL AD(K) BY "CALL AKILL(SAM)".              +
C++++++++++++++++++++++++++++++++++++++++++++++++++
 IF(R(J,K).GT.DCRIT) GO TO 51
 SAM=R(J,K)
 CALL AKILL(SAM)
 KT=KT+1
 CONTINUE
C++++++++++++++++++++++++++++++++++++++++++++++
C+  THE ABOVE "CONTINUE", REPLACED BY "CALL   +
C+  PRINT(24)" WILL GIVE A VERIFICATION       +
C++++++++++++++++++++++++++++++++++++++++++++++
 IF(UN(KT).LT.ACKILL) GO TO 56
 GO TO 51
 56 CONTINUE
C++++++++++++++++++++++++++++++++++++++++++++++
C+  THE ABOVE "56 CONTINUE", WHEN REPLACED BY +
C+  "CALL PRINT(4)", WILL GIVE A VERIFICATION +
C++++++++++++++++++++++++++++++++++++++++++++++
 AD(K)=0
 DO 22 L=1,M
 H(L,K)=0
 22 RTF(L,K)=0
 GO TO 5
C++++++++++++++++++++++++++++++++++++++++++++++++
C+  IF THE SLANT RANGE BETWEEN AC(J) AND AD(K)   +
C+  IS LESS THAN THE MAXIMUM POSSIBLE AIR        +
C+  DEFENSE DETECTION RANGE, AN ACQUISITION IS   +
C+  POSSIBLE.                                    +
```

```
C+++++++++++++++++++++++++++++++++++++++++++++++++
 51 IF(R(J,K).GT.MAXRNG(J,K)) GO TO 52
 CONTINUE
C++++++++++++++++++++++++++++++++++++++++++
C+  THE ABOVE "CONTINUE", REPLACED BY "CALL +
C+  PRINT(9)", RESULTS IN A VERIFICATION.   +
C++++++++++++++++++++++++++++++++++++++++++
 H(J,K)=1
 GO TO 53
C+++++++++++++++++++++++++++++++++++++++++++++
C+ IF THE SLANT RANGE BETWEEN AC(J) AND AD(K)   +
C+ IS GREATER THAN MAXRNG(J,K), NO DETECTION    +
C+ IS POSSIBLE.                                 +
C+                                              +
C+ THE "52 CONTINUE" BELOW, REPLACED BY "CALL   +
C+ PRINT (1)", RESULTS IN A VERIFICATION.       +
C+++++++++++++++++++++++++++++++++++++++++++++
 52 CONTINUE
 H(J,K)=0
 TT(J,K)=0
 53 SUMH=SUMH+H(J,K)
 GO TO 5
 50 CONTINUE
C+++++++++++++++++++++++++++++++++++++++++++++
C+ THE "50 CONTINUE" ABOVE, REPLACED BY "CALL   +
C+ PRINT (3)", RESULTS IN A VERIFICATION.       +
C+++++++++++++++++++++++++++++++++++++++++++++
 5 CONTINUE
C****************************************************
C* AT THIS POINT IT IS KNOWN IF THIS JTH AIRCRAFT IS
C* IN RANGE OF SOME AIR DEFENSE SITE.  FOUR POSSI-
C* BILITIES EXIST:
C*  1) AC(J) ISN'T IN RANGE OF ANY SITE, GO TO "DO 4"
C*     FOR ANOTHER AIRCRAFT.
C*  2) AC(J) IS BEING FIRED UPON AT THIS TIME, GO TO
C*     THE FIRING SECTION.
C*  3) AC(J) IS BEING TRACKED, GO TO THE TRACKING
C*     SECTION TO SEE IF THE MINIMUM TIME IS UP.
C*  4) AC(J) IS IN RANGE OF THE AD FOR THE FIRST TIME
C*     ATTEMPT TO ACQUIRE THE AIRCRAFT.
C****************************************************
 IF(SUMH.EQ.0) GO TO 42
C***************************************************
C*  THE "DO 6" LOOP BELOW DETERMINES THE INTERREL-   *
C*  ATIONSHIPS.                                      *
C*                                                   *
C*  THE "IF(SUMH.EQ.....". ABOVE CHECKS FOR THE      *
C*  AIRCRAFT BEING OUT OF RANGE.                     *
C***************************************************
 DO 6 K=1,N
```

```
C+++++++++++++++++++++++++++++++++++++++++++++++++++++++
C+  THE "IF" FOLLOWING CHECKS FOR THIS AIRCRAFT IN    +
C+  RANGE OF THE KTH AD SITE.  IF NOT IN RANGE, THE   +
C+  NEXT AD IS COMPAIRED.                             +
C+++++++++++++++++++++++++++++++++++++++++++++++++++++++
 IF(H(J,K).EQ.0 ) GO TO 6
C+++++++++++++++++++++++++++++++++++++++++++++++++++++++
C+  THE FOLLOWING "IF" DETERMINES IF AC(J) IS BEING  +
C+  FIRED UPON BY AD(K), AND IF SO GOES TO THE FIRING+
C+  SECTION.                                          +
C+++++++++++++++++++++++++++++++++++++++++++++++++++++++
 IF(F(J,K).EQ.2) GO TO 31
C+++++++++++++++++++++++++++++++++++++++++++++++++++++++
C+  THE FOLLOWING "IF" DETERMINES IF AC(J) IS BEING  +
C+  TRACKED.                                          +
C+++++++++++++++++++++++++++++++++++++++++++++++++++++++
 IF(TT(J,K)-0) 61,61,62
 61 KT=KT+1
C+++++++++++++++++++++++++++++++++++++++++++++++++++++++
C+  THE "61 KT=..." ABOVE INDICATES AC(J) IS NOT BE- +
C+  ING TRACKED, A RANDOM NUMBER WILL BE DRAWN TO     +
C+  DETERMINE IF AD(K) ACQUIRES AC(K) AT THIS TIME.   +
C+                                                    +
C+  THE "CONTINUE" BELOW, REPLACED WITH "CALL         +
C+  PRINT(24)", WILL RESULT IN A VERIFICATION.        +
C+++++++++++++++++++++++++++++++++++++++++++++++++++++++
 CONTINUE
 IF(UN(KT).GT.ACQUIR) GO TO 6001
C++++++++++++++++++++++++++++++++++++++++++++++++++++++
C+  THE ABOVE "IF" COMPARES THE RANDOM NUMBER FOR    +
C+  ACQUISITION.                                     +
C+                                                   +
C+  THE "CONTINUE" FOLLOWING "TT(J,K)..." BELOW,     +
C+  REPLACED BY "CALL PRINT(11)" VERIFIES.           +
C++++++++++++++++++++++++++++++++++++++++++++++++++++++
 TT(J,K)=TIMING
 CONTINUE
 GO TO 6
 62 IF(TT(J,K).GE.CTT) GO TO 64
C++++++++++++++++++++++++++++++++++++++++++++++++++++++
C+  "62 IF(..." INDICATES AD(K) HAS BEEN TRACKING    +
C+  AC(J); IF THE TRACK HAS BEEN MAINTAINED LONG     +
C+  ENOUGH A "READY TO FIRE" INDICATOR WILL BE SET   +
C+  AT "64 RTF(J,K)=K" BELOW.  IF THE TRACK HAS NOT +
C+  BEEN LONG ENOUGH, THE TRACKING TIME WILL BE      +
C+  INCREMENTED AT "63 TT(J,K)=...".                 +
C+                                                   +
C+  THE "CONTINUE" BELOW, REPLACED BY "CALL          +
C+  PRINT(13)", WILL GIVE A VERIFICATION.            +
C++++++++++++++++++++++++++++++++++++++++++++++++++++++
```

```
CONTINUE
63 TT(J,K)=TT(J,K)+TIMING
GO TO 6
64 RTF(J,K)=K
C++++++++++++++++++++++++++++++++++++++++++++++++++
C+  THE "CONTINUE" BELOW, REPLACED BY "CALL      +
C+  PRINT (12)", WILL GIVE A VERIFICATION.       +
C++++++++++++++++++++++++++++++++++++++++++++++++++
CONTINUE
TT(J,K)=TT(J,K)+TIMING
GO TO 6
6001 CONTINUE
C++++++++++++++++++++++++++++++++++++++++++++++++++
C+ THE "6001 CONTINUE" ABOVE, REPLACED BY "CALL  +
C+  PRINT(10)", WILL GIVE A VERIFICATION.        +
C++++++++++++++++++++++++++++++++++++++++++++++++++
6 CONTINUE
C****************************************************
C*  AT THIS POINT IT IS KNOWN WHICH SITES ARE NOW  *
C*  "READY TO FIRE"; THE CLOSEST SITE NOT CURRENT- *
C*  LY ENGAGING AN AIRCRAFT WILL BE SELECTED TO    *
C*  ATTEMPT TO LAUNCH A MISSILE.  THE "DO 7" BELOW *
C*  DETERMINES THE RANGE TO AC(J) FOR ALL AD(K).   *
C****************************************************
DO 7 K=1,N
IF(RTF(J,K).EQ.0) GO TO 71
IF(F(J,K).NE.0) GO TO 71
FRNG(K)=R(J,K)
GO TO 72
71 FRNG(K)=99999
72 CONTINUE
C++++++++++++++++++++++++++++++++++++++++++++
C+  THE "72 CONTINUE" ABOVE, REPLACED WITH +
C+  "CALL PRINT(20)", WILL GIVE A VERIFICA-+
C+  TION PRINT OUT.                        +
C++++++++++++++++++++++++++++++++++++++++++++
7 CONTINUE
SMALL=AMIN1(FRNG(1),FRNG(2),FRNG(3),FRNG(4))
IF(SMALL.EQ.99999) GO TO 43
C***************************************************
C*  THE "DO 10" LOOP WILL SELECT THE CLOSEST AD   *
C*  SITE, ATTEMPT A LAUNCH (AT "12 KT=..."), CHECK*
C*  THE SUCCESS, AND TAKE THE NECESSARY ACTION.   *
C***************************************************
DO 10 K=1,N
IF(SMALL.NE.R(J,K)) GO TO 10
DO 8 L=1,M
8 F(L,K)=1
CONTINUE
C+++++++++++++++++++++++++++++++++++++++++++++++
```

```
C+  THE ABOVE "CONTINUE", REPLACED BY "CALL   +
C+  PRINT(15)", WILL GIVE A VERIFICATION.     +
C++++++++++++++++++++++++++++++++++++++++++++++++
 12 KT=KT+1
 CONTINUE
C++++++++++++++++++++++++++++++++++++++++++++++++
C+  THE ABOVE "CONTINUE", REPLACED BY "CALL   +
C+  PRINT(24)", WILL GIVE A VERIFICATION.     +
C++++++++++++++++++++++++++++++++++++++++++++++++
 IF(UN(KT).LT.GOOD) GO TO 13
 CONTINUE
C++++++++++++++++++++++++++++++++++++++++++++++++
C+  THE ABOVE "CONTINUE", REPLACED BY "CALL   +
C+  PRINT(17)", WILL GIVE A VERIFICATION.     +
C++++++++++++++++++++++++++++++++++++++++++++++++
 TT(J,K)= TT(J,K)-RECT
 DO 81 L=1,M
 F(L,K)=0
 81 RTF(L,K)=0
 10 CONTINUE
 GO TO 4
C****************************************************
C*  "13 F(J,K)=2" INDICATES A SUCCESSFUL MISSILE  *
C*  LAUNCH; THE INTERCEPT TIME IS COMPUTED, AND  *
C*  CAN BE PRINTED BY REPLACING THE FOLLOWING    *
C*  "CONTINUE" WITH "CALL PRINT(16)".            *
C****************************************************
 13 F(J,K)=2
 TI(J,K)=IT+R(J,K)/(ACVOL(J)+ADMSLSP)
 CONTINUE
 GO TO 4
C****************************************************
C*  "13 IF(IT.GE....)" BELOW INDICATES A MISSILE  *
C*  HAS BEEN FIRED ON AC(J); IF IT IS TOO EARLY   *
C*  FOR AN INTERCEPT, GO TO THE NEXT AC; IF IT IS *
C*  TIME FOR AN INTERCEPT GO TO "32 KT=...".      *
C*                                                *
C*  THE "CONTINUE" FOLLOWING MAY BE REPLACED BY   *
C*  "CALL PRINT(21)" FOR A VERIFICATION .         *
C****************************************************
 31 IF(IT.GE.TI(J,K)) GO TO 32
 CONTINUE
 GO TO 4
 32 KT=KT+1
 CONTINUE
C++++++++++++++++++++++++++++++++++++++++++++++++
C+  THE ABOVE "CONTINUE", REPLACED BY "CALL    +
C+  PRINT(24)", WILL GIVE A VERIFICATION.      +
C+                                             +
C+  THE "IF(UN(..." FOLLOWING DETERMINES THE   +
```

```
C+  KILL OF THE AIRCRAFT BY THE MISSILE.  IF   +
C+  THERE WAS A KILL, GO TO "33 AC(J...", IF   +
C+  THERE WAS NO KILL, GO BACK TO "12".        +
C+                                             +
C+  THE "CONTINUE" FOLLOWING, REPLACED BY      +
C+  "CALL PRINT(19)", WILL GIVE A VERIFICATION.+
C+++++++++++++++++++++++++++++++++++++++++++++++
 IF(UN(KT).LT.ADKILL) GO TO 33
 CONTINUE
 F(J,K)=1
 GO TO 12
 33 AC(J)=0
 DO 9 JI=1,M
 F(JI,K)=0
 RTF(JI,K)=0
 TT(JI,K)=0
 9 CONTINUE
 CONTINUE
C++++++++++++++++++++++++++++++++++++++++++++++
C+  THE ABOVE "CONTINUE", REPLACED BY  "CALL  +
C+  PRINT(18)", WILL GIVE A VERIFICATION.     +
C++++++++++++++++++++++++++++++++++++++++++++++
 GO TO 4
C+++++++++++++++++++++++++++++++++++++++++++++++
C+  THE "40 CONTINUE", "41 CONTINUE", "42 CON-  +
C+  TINUE", AND "43 CONTINUE" MAY BE REPLACED   +
C+  BY "CALL PRINT (5),(6),(2),(14)" RESPECTIVE-+
C+  LY FOR A VERIFICATION PRINT OUT.            +
C+++++++++++++++++++++++++++++++++++++++++++++++
 40 CONTINUE
 GO TO 4
 41 CONTINUE
 GO TO 4
 42 CONTINUE
 GO TO 4
 43 CONTINUE
 4 CONTINUE
C**********************************************
C*  ALL AIRCRAFT HAVE BEEN CHECKED FOR THIS TIME   *
C*  PERIOD.  INSERTING "CALL PRINT (22)" AND "CALL *
C*  PRINT(23)" AFTER "NI=N" WILL PRODUCE A VERIFI- *
C*  CATION PRINT OUT.                              *
C*                                                 *
C*  THE "IF(SUMJ.EQ. ... " WILL END THIS REPETITION*
C*  OF THE BATTLE IF ALL COMBATANTS ARE KILLED.    *
C**************************************************
 NI=N
 SUMJ=0
 DO 105 KILL=1,M
 105 SUMJ=SUMJ+AC(KILL)
```

```
 SUMK=0
 DO 106 KIL=1,N
 106 SUMK=SUMK+AD(KIL)
 IF((SUMJ.EQ.0).OR.(SUMK.EQ.0))GO TO 999
C++++++++++++++++++++++++++++++++++++++++++++++++
C+  THE TIME IS INCREMENTED HERE                 +
C++++++++++++++++++++++++++++++++++++++++++++++++
 IT=IT+TIMING
C++++++++++++++++++++++++++++++++++++++++++++++++
C+  "DO 70" MOVES ALL SURVIVING AIRCRAFT.        +
C++++++++++++++++++++++++++++++++++++++++++++++++
 DO 70 J=1,M
 A(J)=A(J)-ACVOL(J)*TIMING
 SUMK=0
 70 CONTINUE
C++++++++++++++++++++++++++++++++++++++++++++++++
C+  "GO TO 20" SENDS AIRDEF BACK TO THE START OF +
C+  A NEW TIME PERIOD.                           +
C++++++++++++++++++++++++++++++++++++++++++++++++
 GO TO 20
 999 DO 102 J=1,M
C*************************************************
C*  "SUMAC(J)" AND "SUMAD(K)" ACCUMULATE THE ENDING*
C*  STATUS FOR ALL COMBATANTS.                    *
C*************************************************
 102 SUMAC(J)=SUMAC(J)+AC(J)
 DO 1021 K=1,N
 1021 SUMAD(K)=SUMAD(K)+AD(K)
 CALL OAS(1)
C*************************************************
C*  "CALL OAS(1)" ABOVE ACCUMULATES THE NUMBER OF  *
C*  SITES SURVIVING THIS BATTLE.                   *
C*                                                 *
C*  "101 CONTINUE" BELOW SENDS AIRDEF BACK TO THE  *
C*  INITIALIZATION PORTION TO START ANOTHER BATTLE *
C*                                                 *
C*  "DO 103" COMPUTES "AVEAD(K)" AND PRINTS BOTH   *
C*  THE PROBABILITY OF SURVIVAL FOR EACH SITE AND  *
C*  THE BARTLETT TRANSFORMATION, IF DESIRED.       *
C*************************************************
 101 CONTINUE
 DO 103 K=1,N
 AVEAD(K)=SUMAD(K)/LIMIT
 TRANS=SQRT(AVEAD(K))
 FORM=ASIN(TRANS)
 WRITE(6,104) K,AVEAD(K)
 104 FORMAT(*SURVIVAL PROBABILITY FOR AIR DEFENSE *
+*SITE (*,I1,*) =*,F6.4)
 103 CONTINUE
 SYS=0
```

```
C*************************************************
C*  "DO 107" COMPUTES THE EXPECTED  PROB-        *
C*  ABILITY OF SURVIVAL BASED UPON AN EQUALLY    *
C*  LIKELY DIRECTION OF ATTACK.  THE BARTLETT    *
C*  TRANSFORMATION IS ALSO COMPUTED FOR REFERENCE*
C*************************************************
 DO 107 K=1,N
 107 SYS=SYS+AVEAD(K)
 PSYS=SYS/4
 TRANS=SQRT(PSYS)
 FORM=ASIN(TRANS)
 WRITE(6,108) PSYS
 108 FORMAT(/,23X,*EXPECTED SURVIVABILITY = *,F6.4,/)
 CALL OAS(5)
C***************************************************
C*  "CALL OAS(5)" ABOVE PRINTS THE NUMBER OF TIMES *
C*  0,1,2,3,OR 4 SITES SURVIVED.                   *
C*                                                 *
C*  "DO 201" BELOW SAVES THE AVEAD(K) FOR EACH     *
C*  ESTIMATE FOR USE IN "DO 202", "DO 203", AND    *
C*  "DO 204" BELOW.                                *
C***************************************************
 DO 201 K=1,N
 201 AAD(LOOPS,K)=AVEAD(K)
 200 CONTINUE
C***************************************************
C*  "200 CONTINUE" ENDS THE ESTIMATE GENERATION;   *
C*  "ADMEAN(K)" COMPUTED BELOW IS THE AVERAGE FOR  *
C*  EACH AIR DEFENSE SITE FOR ALL THE BATTLES.     *
C***************************************************
 DO 202 K=1,N
 DO 203 J=1,IN
 SUMAAD(K)=SUMAAD(K)+AAD(J,K)
 SUMSQR(K)=SUMSQR(K)+AAD(J,K)**2
 203 CONTINUE
 202 CONTINUE
 DO 204 K=1,N
 ADMEAN(K)=SUMAAD(K)/IN
 VAR(K)=(SUMSQR(K)-((SUMAAD(K))**2)/IN)/(IN-1)
 WRITE(6,205)K,ADMEAN(K),VAR(K)
 205 FORMAT(*  MEAN FOR AD(*,I1,*)=*,F6.4,*  VARIANCE=
     *  *,F6.4)
 204 CONTINUE
 GRNMN=0
 DO 206 K=1,N
 206 GRNMN=GRNMN+ADMEAN(K)
 GRNMA=GRNMN/N
 PRINT(6,*)"OVERALL ADSSP ESTIMATE IS ",GRNMA
 STOP
 END
```

```
SUBROUTINE PRINT(N)
C*********************************************
C*  THIS SUBROUTINE CONTAINS ALL THE PRINT   *
C*  STATEMENTS FOR THE VERIFICATION VERSION  *
C*  OF AIRDEF.  THE VERIFICATION "CALL PRINT"*
C*  STATEMENTS SHOULD BE INSERTED ONLY WHEN  *
C*  "IN" AND "LIMIT" ARE BOTH EQUAL TO 0.    *
C*********************************************
COMMON/1/AC(12),AD(4),INT(12),D(12,4),R(12,4),H(12,4)
COMMON/2/TT(12,4),FRNG(4),RTF(12,4),TI(12,4),F(12,4),
*  A(12)
COMMON/3/B(12),C(12),X(4),Y(4),Z(4),ACVOL(12),UN(500)
COMMON/4/MAXRNG(12,4),J,K,M,NI,SUMJ,SUMK,KT,IT
 IF(N.EQ.1)GO TO 1
 IF(N.EQ.2) GO TO 2
 IF(N.EQ.3) GO TO 3
 IF(N.EQ.4) GO TO 4
 IF(N.EQ.5) GO TO 5
 IF(N.EQ.6) GO TO 6
 IF(N.EQ.7) GO TO 7
 IF(N.EQ.8) GO TO 8
IF(N.EQ.9) GO TO 9
IF(N.EQ.10) GO TO 10
 IF(N.EQ.11 ) GO TO 11
 IF(N.EQ.12) GO TO 12
 IF(N.EQ.13) GO TO 13
IF(N.EQ.14) GO TO 14
 IF(N.EQ.15) GO TO 15
 IF(N.EQ.16) GO TO 16
 IF(N.EQ.17) GO TO 17
IF(N.EQ.18) GO TO 18
IF(N.EQ.19) GO TO 19
IF(N.EQ.20) GO TO 20
IF(N.EQ.21) GO TO 21
IF(N.EQ.22)GO TO 22
IF(N.EQ.23) GO TO 23
IF(N.EQ.24) GO TO 24
IF(N.EQ.25) GO TO 25
1 WRITE(6,100) K
100 FORMAT(*RANGE TOO GREAT FOR DETECTION BY AD(*,I1,
*  *)*)
GO TO 99
2 WRITE(6,200)J
200 FORMAT(*AC(*,I1,*) IS NOT IN RANGE OF ANY SITE*)
GO TO 99
3 WRITE(6,300)K
300 FORMAT(*AD(*,I1,*) IS NOT IN ACTION*)
GO TO 99
4 WRITE(6,400)J,K
400 FORMAT(*AC(*,I1,*) HAS KILLED AD(*,I1,*)*)
```

```
GO TO 99
5 WRITE(6,500)J
500 FORMAT(*AC(*,I1,*) HAS ALREADY BEEN KILLED*)
GO TO 99
6 WRITE(6,600)J
600 FORMAT(*AC(*,I1,*) HAS NOT ENTERED THE BATTLE*)
GO TO 99
7 WRITE(6,700)J
700 FORMAT(/,*FOR AC(*,I1,*)*,25(*.*),/)
GO TO 99
8 WRITE(6,800) J,K,R(J,K)
800 FORMAT(*RANGE BETWEEN AC(*,I1,*) AND AD(*,I1,*)
     *  IS*,F15.2)
GO TO 99
9 WRITE(6,900) J,K
900 FORMAT(*DETECTION OF  AC(*,I1,*) BY  AD(*,I1,*)
     *  IS   POSSIBLE*
GO TO 99
10 WRITE(6,1000)J,K
1000 FORMAT(*AC(*,I1,*) IS NOT ACQUIRED BY AD(*,I1,*)
     *  *)
GO TO 99
11 WRITE(6,1100) J,K
1100 FORMAT(*AC(*,I1,*) IS ACQUIRED BY AD(*,I1,*)*)
GO TO 99
12 WRITE(6,1200)J,K
1200 FORMAT(*AC(*,I1,*) HAS BEEN TRACKED LONG ENOUGH
     *  BY *
+*AD(*,I1,*) AND IS READY TO FIRE*)
GO TO 99
13 WRITE(6,1300) J,K
1300 FORMAT(*AC(*,I1,*) HAS NOT BEEN TRACKED LONG
     *  ENOUGH BY *
+*AD(*,I1,*) *)
GO TO 99
14 WRITE(6,1400)
1400 FORMAT(*NO SITES ARE REACY TO FIRE*)
GO TO 99
15 WRITE(6,1500) K
1500 FORMAT(*AD(*,I1,*) WILL ATTEMPT TO FIRE*)
GO TO 99
16 WRITE(6,1600)K,TI(J,K)
1600 FORMAT(*AD(*,I1,*) WILL INTERCEPT AT*,F6.2)
GO TO 99
17 WRITE(6,1700) K,TT(J,K)
1700 FORMAT(*AD(*,I1,*) FIRING UNSUCCESSFUL, ATTEMPT
     *  TO FIRE AGAIN
GO TO 99
18 WRITE(6,1800) J,K
1800 FORMAT(*AC(*,I1,*) KILLED BY AD(*,I1,*)*)
```

```
GO TO 99
19 WRITE (6,1900) J,K
1900 FORMAT(*AC(*,I1,*) NOT KILLED BY AD(*,I1,*)*)
GO TO 99
20 WRITE(6,2000)J,K,RTF(J,K),J,K,F(J,K),K,FRNG(K)
2000FORMAT(*RTF(*,I1,1H,,I1,*)=*,F3.0,*; F(*,I1,1H,,
     *  I1,*)=*,F3.0,*
+* FRNG(*,I1,*)=*,F9.2)
GO TO 99
21 WRITE(6,2100) K
2100 FORMAT(*AD(*,I1,*) IS ENGAGING THIS AIRCRAFT*)
GO TO 99
22 SUMJ=0
DO 602 JM=1,M
SUMJ=SUMJ+AC(JM)
IF(AC(JM).EQ.0) GO TO 621
WRITE(6,603) JM
603 FORMAT(3X,*AIRCRAFT*,I3,* IS STILL ALIVE*)
GO TO 602
621 WRITE(6,622) JM
622 FORMAT(3X,*AIRCRAFT*,I3,* HAS BEEN KILLED*)
602 CONTINUE
GO TO 99
23 SUMK=0
DO 604 K=1,NI
SUMK=SUMK+AD(K)
IF((AD(K)).EQ.1) GO TO 641
WRITE(6,605) K
605 FORMAT(3X,*AIR DEFENSE SITE*,I3,*  HAS BEEN
     *  KILLED*)
GO TO 604
641 WRITE(6,642) K
642 FORMAT(3X,*AIR DEFENSE SITE*, I3,*  STILL IN
     *  ACTION*)
604 CONTINUE
GO TO 99
24 WRITE(6,2400)KT,UN(KT)
2400 FORMAT(*UN(*,I3,*)=*,F6.4)
GO TO 99
25 WRITE(6,2500) IT
2500 FORMAT(/,50(1H*),//,3X,*CURRENT TIME IS *,F6.2,//
     *  ,50(1H*))
GO TO 99
99 RETURN
END
SUBROUTINE AKILL(SAM)
COMMON/5/ACKILL
C*********************************************
C*  THIS SUBROUTINE DETERMINES THE AIRCRAFT KILL*
C*  PROBABILITIES AT VARIOUS RANGES.            *
```

```
C*************************************************
 IF(SAM.GT.40.) ACKILL=.90
 IF((SAM.LE.40.).AND.(SAM.GT.30.)) ACKILL=.90
 IF((SAM.LE.30.).AND.(SAM.GT.20.)) ACKILL=.90
 IF((SAM.LE.20.).AND.(SAM.GT.10.)) ACKILL=.90
 IF((SAM.LE.10.)) ACKILL=.90
 RETURN
 END
 SUBROUTINE DAS(NUMBER)
C*************************************************
C*  THIS SUBROUTINE ACCUMULATES THE NUMBER OF    *
C*  AIR DEFENSE SITES SURVIVING EACH BATTLE.     *
C*                                               *
C*  IF "CALL DAS(5)" IS UTILIZED, THEN THE       *
C*  NUMBER OF SURVIVING SITES IS PRINTED.        *
C*************************************************
 COMMON/4/MAXRNG(12,4),J,K,M,NI,SUMJ,SUMK,KT,IT
 COMMON/7/DEF0,DEF1,DEF2,DEF3,DEF4,LIMIT,PC0,PC1,PC2,
*   PC3,PC4
 IF(NUMBER.EQ.5) GO TO 5
 IF(SUMK.EQ.0) DEF0=DEF0+1
 IF(SUMK.EQ.1) DEF1=DEF1+1
 IF(SUMK.EQ.2) DEF2=DEF2+1
 IF(SUMK.EQ.3) DEF3=DEF3+1
 IF(SUMK.EQ.4) DEF4=DEF4+1
 GO TO 99
 5 CONTINUE
 PC0=DEF0/LIMIT
 PC1=DEF1/LIMIT
 PC2=DEF2/LIMIT
 PC3=DEF3/LIMIT
 PC4=DEF4/LIMIT
 WRITE(6,10)DEF0,PC0
 10 FORMAT(*NUMBER TIMES 0 SITES SURVIVED =*,F5.1,* ;
*  % = *,F6.4)
 WRITE(6,20)DEF1,PC1
 20 FORMAT(*NUMBER TIMES 1 SITE  SURVIVED =*,F5.1,* ;
*  % = *,F6.4)
 WRITE(6,30) DEF2,PC2
 30 FORMAT(*NUMBER TIMES 2 SITES SURVIVED =*,F5.1,* ;
*  % = *,F6.4)
 WRITE(6,40) DEF3,PC3
 WRITE(6,50) DEF4,PC4
 40 FORMAT(*NUMBER TIMES 3 SITES SURVIVED =*,F5.1,* ;
*  % = *,F6.4)
 50 FORMAT(*NUMBER TIMES 4 SITES SURVIVED =*,F5.1,* ;
*  % = *,F6.4,/)
 99 RETURN
 END
```

APPENDIX B

This appendix contains a complete BASIC listing of the regression equation along with an example of its use. The program is entirely interactive; input is made in free-field format. The program listing follows the example of its use.

```
TYPE IN THE NUMBER ØF ATTACKING AIRCRAFT
? 6
ØRIGINAL X1 IS  . . . . . 6
TRANSFØRMED X1 IS . . . . 1.5
 TYPE IN THE ATTACK SPEED ØF THE AIRCRAFT
? 750
ØRIGINAL X2 IS  . . . . . 750
TRANSFØRMED X2 IS . . . . 2.2
TYPE IN THE MAXIMUM KILL RANGE ØF THE AIRCRAFT
? 65
ØRIGINAL X3 IS  . . . . . 65
TRANSFØRMED X3 IS . . . . 2.16667
TYPE IN THE AIR DEFENSE ACQUISITIØN CAPABILITY (IN %)
? .65
ØRIGINAL X4 IS  . . . . . .65
TRANSFØRMED X4 IS . . . . 1.8571

 THE X VECTØR IS
 1              7.15           6.4048         8.85218        2.25
 1.5            4.76667        4.69444        2.16667

THE ESTIMATED AIR DEFENSE SURVIVABILITY IS .687239

WITH 99% CERTAINTY THAT THE TRUE RATE IS BETWEEN

 .619439      AND           .755039

MØRE ESTIMATES ?  IF YES TYPE 1 ; IF NØ TYPE 0? 0

SRU      0.388 UNTS.

RUN CØMPLETE.
```

```
00100 DIM B(1,9),X(9,1),Y(1,1)
00110 DIM X1(1,9)
00120 B(1,1)=.733333
00130 B(1,2)=-.202692
00140 B(1,3)=-.010332
00150 B(1,4)=.0112664
00160 B(1,5)=-.17756
00170 B(1,6)=.930932
00180 B(1,7)=.109305
00190 B(1,8)=-.0547531
00200 B(1,9)=.184061
00210 PRINT"TYPE IN THE NUMBER OF ATTACKING AIRCRAFT"
00220 INPUT X1
00230 PRINT "ORIGINAL X1 IS  . . . ."; X1
00240 X1=X1/4
00250 PRINT "TRANSFORMED X1 IS . . ."; X1
00260 PRINT " TYPE IN THE ATTACK SPEED OF THE AIRCRAFT"
00270 INPUT X2
00280 PRINT "ORIGINAL X2 IS  . . . ."; X2
00290 X2=(1+.004*(X2-450))
00300 PRINT "TRANSFORMED X2 IS . . ."; X2
00310 PRINT "TYPE IN THE MAXIMUM KILL RANGE OF THE
00320 INPUT X3
00330 PRINT "ORIGINAL X3 IS  . . . ."; X3
00340 X3=X3/30
00350 PRINT "TRANSFORMED X3 IS . . ."; X3
00360 PRINT "TYPE IN THE AIR DEFENSE ACQUISITION CAPABILITY
     *  (IN %)"
00370 INPUT X4
00380 PRINT "ORIGINAL X4 IS  . . . ."; X4
00390 X4=(1+2.857*(.95-X4))
00400 PRINT "TRANSFORMED X4 IS . . ."; X4
00410 X(1,1)=1
00420 X(2,1)=X1*X2*X3
00430 X(3,1)=X4**3
00440 X(4,1)=X2*X3*X4
00450 X(5,1)=X1**2
00460 X(6,1)=X1
00470 X(7,1)=X2*X3
00480 X(8,1)=X3**2
00490 X(9,1)=X3
00500 MAT Y=B*X
00510 Y1=Y(1,1)
00520 IF Y1<0 THEN 00690
00530 Y2=(SIN(Y1))**2
00540 Y3=Y2+.0678
00550 Y4=Y2-.0678
00560 PRINT
00570 PRINT " THE X VECTOR IS "
00580 MAT X1=TRN(X)
```

```
00590 MAT PRINT X1
00600 PRINT"THE ESTIMATED AIR DEFENSE SURVIVABILITY IS";Y2
00610 PRINT
00620 PRINT"WITH 99% CERTAINTY THAT THE TRUE RATE IS
     *  BETWEEN"
00630 PRINT
00640 PRINT Y4,"AND",Y3
00650 PRINT
00660 PRINT"MORE ESTIMATES ?  IF YES TYPE 1 ; IF NO TYPE
00670 INPUT K1
00680 IF K1>0 THEN 00210
00685 IF K1=0 THEN 00710
00690 Y1=0
00700 IF Y1=0 THEN 00530
00710 STOP
00720 END
```

APPENDIX C

This appendix contains a complete BASIC listing of the step-wise regression program used to find the regression equation. Also included is the regression equation prediction and associated confidence interval on the true mean value of survivability at each of the 78 specific aircraft attack conditions. The program listing follows the regression predictions.

ESTIMATED Y? IF YES, TYPE 1; ELSE 0? 1

ID	P-HAT	P-ØBS	99% CØNFIDENCE INTERVAL LØWER LIMIT	UPPER LIMIT
1	.746403	.750002	.697481	.792385
2	.746403	.706709	.697481	.792385
3	.746403	.646711	.697481	.792385
4	.74228	.783312	.702695	.77997
5	.74228	.686669	.702695	.77997
6	.74228	.776685	.702695	.77997
7	.622947	.693328	.571891	.67267
8	.622947	.716674	.571891	.67267
9	.622947	.640005	.571891	.67267
10	.820085	.803311	.767478	.867269
11	.820085	.843301	.767478	.867269
12	.820085	.840015	.767478	.867269
13	.816441	.856653	.769848	.85884
14	.816441	.810024	.769848	.85884
15	.816441	.776685	.769848	.85884
16	.707458	.609897	.648531	.763084
17	.707458	.723319	.648531	.763084
18	.707458	.690003	.648531	.763084
19	.511294	.506702	.428295	.593981
20	.511294	.490003	.428295	.593981
21	.511294	.596689	.428295	.593981
22	.540325	.483305	.480851	.599228
23	.540325	.543347	.480851	.599228
24	.540325	.533377	.480851	.599228
25	.445567	.413342	.368055	.524435
26	.445567	.500002	.368055	.524435
27	.445567	.426667	.368055	.524435
28	.7775	.730004	.724632	.826214
29	.7775	.696643	.724632	.826214
30	.7775	.746704	.724632	.826214
31	.744662	.736637	.691652	.794272
32	.744662	.730004	.691652	.794272
33	.744662	.733371	.691652	.794272
34	.592611	.6767	.527552	.656079
35	.592611	.636641	.527552	.656079
36	.592611	.609995	.527552	.656079
37	.598427	.609995	.541412	.654138
38	.598427	.636641	.541412	.654138
39	.598427	.640005	.541412	.654138
40	.57157	.559957	.516149	.626104
41	.57157	.673328	.516149	.626104
42	.57157	.436672	.516149	.626104
43	.420908	.406653	.35911	.483976
44	.420908	.433301	.35911	.483976
45	.420908	.359999	.35911	.483976
46	.832752	.873322	.784237	.876197

47	.832752	.833362	.784237	.876197
48	.832752	.850002	.784237	.876197
49	.803039	.829994	.75357	.848203
50	.803039	.790031	.75357	.848203
51	.803039	.816648	.75357	.848203
52	.660178	.673328	.596004	.721527
53	.660178	.683324	.596004	.721527
54	.660178	.590008	.596004	.721527
55	.787604	.782983	.735877	.835085
56	.787604	.810024	.735877	.835085
57	.787604	.786681	.735877	.835085
58	.745517	.703334	.694459	.793398
59	.745517	.773344	.694459	.793398
60	.745517	.796671	.69459	.793398
61	.582485	.569971	.515028	.648426
62	.582485	.569971	.515028	.648426
63	.582485	.623316	.515028	.648426
64	.771185	.753372	.721794	.817076
65	.771185	.816648	.721794	.817076
66	.771185	.793361	.721794	.817076
67	.728079	.706617	.679578	.774006
68	.728079	.726623	.679578	.774006
69	.728079	.709164	.679578	.774006
70	.562895	.543347	.496266	.628403
71	.562895	.529984	.496266	.628403
72	.562895	.510001	.496266	.628403
73	.304484	.320033	.246679	.365566
74	.304484	.353293	.246679	.365566
75	.304484	.353293	.246679	.365566
76	.269642	.213322	.214802	.328263
77	.269642	.256699	.214802	.328263
78	.269642	.240023	.214802	.328263

```
00050 FILE #1 = "KOH"
100 DIM O(1,20)
110 DIM Y(20,20),X(20),S(20),P(20),L(20),E(20)
120 DIM D(400),B(20),F(20),G(5),H(11),U(20),T(20)
125 DIM A(1,9),A1(9,1),C1(9,9),C2(1,9),C4(9,9)
126 DIM C3(1,1)
127 DIM C(15,15)
130 PRINT"STEPWISE REGRESSION MAXIMUM 20 VARIABLES"
140 PRINT "FILE MANAGEMENT MODE  SEE DATA  FILE
150 READ N
160 REM N=NO OF OBSERVATION   M=NO OF VARIABLES
170 READ M
175 M1=M-1
190 PRINT "THIS DATUM HAS "N" OBSERVATIONS AND "M"
     *  VARIABLES"
210 S=N*M
220MAT X=ZER(M)
222MAT Y=ZER(M,M)
230MAT F=ZER(M)
232MAT B=ZER(M)
234MAT O=ZER(M*M)
236MAT U=ZER(M)
260 MAT E=ZER(M)
270 REM   DATA  ENTRY
280 FOR K1=1 TO N
290 MAT READ B
300MAT WRITE #1, B
310 FOR I=1 TO M
320 X(I)=X(I)+B(I)
330 FOR J=1 TO M
340 Y(I,J)=Y(I,J)+B(I)*B(J)
350 NEXT J
360 NEXT I
370 NEXT K1
371 MAT C=IDN(M,M)
372 FOR I=2 TO M
373 FOR J=2 TO M
374 M2=I-1
375 M3=J-1
376 C(I,J)= Y(M2,M3)
377 NEXT J
378 NEXT I
379 C(1,1)=N
380RESTORE #1
390PRINT"LIST OF DATA ? IF YES, TYPE 1; ELSE 0";
400 INPUT M9
410 IF M9=0 THEN  480
420 REM
430 FOR I=1 TO N
440MAT READ #1, B
```

```
442FOR J=1 TO M
444PRINT B(J),
446NEXTJ
450PRINT
460 NEXT I
470RESTORE #1
480 GOSUB 2880
490 PRINT"ID      MEAN"
500 FOR I=1 TO M
510 O=X(I)/N
520 B(I)=O
530 PRINTI;O
540 FOR J=1 TO M
550 Y(I,J)=Y(I,J)-O*X(J)
560 NEXT J
570 NEXT I
571 FOR I1=2 TO M
572 M5=I1-1
573 C(I1,1)=X(M5)/N
574 C(1,I1)=X(M5)/N
575 NEXT I1
581 PRINT
590 PRINT"ROW   COL","CORRECTED SS","SIMPLE CORRELATION
600 FOR I=1 TO M
610 X(I)=X(I)/N
620 FOR J=I TO M
630 R=Y(I,I)*Y(J,J)
640 IF R=0 THEN 660
650 R=Y(I,J)/SQR(R)
660 PRINT I;J,Y(I,J),R
680 NEXT J
690 NEXT I
700 GOSUB 2880
710 PRINT"THE FOLLOWING DOES THE STEPWISE REGRESSION"
720 PRINT"THE APPROPRIATE CODE FOR EACH VARIABLE"
730 PRINT"0 - INDEP. VAR. AVAILABLE FOR SELECTION"
740 PRINT"1 - INDEP. VAR. TO BE FORCED IN REGRESSION"
750 PRINT"2 - VAR. TO BE DELETED"
760 PRINT"3 - DEPENDENT VAR."
770 PRINT
780 PRINT"TYPE THE APPROPRIATE CODE FOR EACH VARIABLE"
782 MAT O=ZER(1,M)
790 MAT INPUT O
792 FOR I=1 TO M
794 F(I)=O(1,I)
796 NEXT I
810 N0=0
820 FOR I=1 TO M
830 FOR J=1 TO M
840 N0=N0+1
```

```
850 D(N0)=Y(I,J)
860 NEXT J
870 NEXT I
880 PRINT"TYPE THE CONSTANT VALUE OF PROPORTION"
890 PRINT"OF SS THAT WILL BE USED TO LIMIT VAR."
900 PRINT"ENTERING THE REG. IF NON, TYPE 0";
910 INPUT P1
920 GOSUB 2880
930 I2=N2=N3=G(3)=H(3)=H(4)=0
940 O=N-1
1000 FOR I=1 TO M
1010 U(I)=1
1020 IF F(I)<=0 THEN 1160
1030 IF F(I)-2>0 THEN 1110
1040 IF F(I)-2=0 THEN 1080
1050 N2=N2+1
1060 F(N2)=I
1070 GOTO 1160
1080 G(3)=G(3)+1
1090 U(I)=-1
1100 GOTO 1160
1110 M1=I
1120 G(1)=M1
1130 L2=M*(M1-1)
1140 L3=L2+M1
1150 H(5)=D(L3)
1160 NEXT I
1170 G(2)=N2
1180 K1=M-G(3)-1
1190MAT S=ZER(K1)
1192MAT P=ZER(K1)
1194MAT L=ZER(K1)
1196MAT T=ZER(K1)
1200 REM START SELECTION OF VARIABLES
1210 FOR N4=1 TO K1
1220 R2=0
1230 IF N4-N2>0 THEN 1350
1240 FOR I=1 TO N2
1250 K=F(I)
1260 IF U(K)<=0 THEN 1330
1270 L3=L2+K
1280 I3=M*(K-1)+K
1290 R1=D(L3)*D(L3)/D(I3)
1300 IF R2-R1>=0 THEN 1330
1310 R2=R1
1320 N5=K
1330 NEXT I
1340 GOTO 1450
1350 FOR I= 1 TO M
1360 IF I-M1=0 THEN 1440
```

```
1370 IF U(I)<=0 THEN1440
1380 L3=L2+I
1390 I3=M*(I-1)+I
1400 R1=D(L3)*D(L3)/D(I3)
1410 IF R2-R1>=0 THEN 1440
1420 R2=R1
1430 N5=I
1440 NEXT I
1450 IF R2<=0 THEN 1470
1460 IF H(5)-(H(3)+R2)>0 THEN1490
1470 I2=1
1480 GOTO 2580
1490 R1=R2/H(5)
1500 IF R1-P1<0 THEN 2580
1510 U(N5)=0
1520 L(N4)=N5
1530 H(1)=R2
1540 H(2)=R1
1550 H(3)=H(3)+R2
1560 H(4)=H(4)+R1
1570 G(4)=N4
1580 G(5)=N5
1590 H(6)=SQR(H(4))
1600 R2=N4
1610 R1=0-R2
1620 R1=(H(5)-H(3))/R1
1630 H(7)=(H(3)/R2)/R1
1640 H(8)=SQR(R1)
1650 I3=M*(N5-1)+N5
1660 R2=D(I3)
1670 L3=N5-M
1680 FOR J= 1 TO M
1690 L3=L3+M
1700 IF U(J)<0 THEN 1780
1710 IF U(J)>0 THEN 1750
1720 IF J-N5=0 THEN 1770
1730 I4=M*(J-1)+J
1740 D(I4)=D(I4)+D(L3)*D(L3)/R2
1750 D(L3)=D(L3)/R2
1760 GOTO 1780
1770 D(I3)=1./R2
1780 NEXT J
1790 L3=L2+N5
1800 P(N4)=D(L3)
1810 IF N4-1 <=0 THEN 1950
1820 I5=N4-1
1830 FOR J=1 TO I5
1840 I4=N4-J
1850 K2=L(I4)
1860 L3=L2+K2
```

```
1870 P(I4)=D(L3)
1880 FOR K=1 TO J
1890 I6=N4-K+1
1900 M2=L(I6)
1910 L3=M*(M2-1)+K2
1920 P(I4)=P(I4)-D(L3)*P(I6)
1930 NEXT K
1940 NEXT J
1950 H(9)=B(M1)
1960 FOR I=1 TO N4
1970 K2=L(I)
1980 H(9)=H(9)-P(I)*B(K2)
1990 I4=M*(K2-1)+K2
2000 S(I)=H(8)*SQR(D(I4))
2010 T(I)=P(I)/S(I)
2020 NEXT I
2030 I3=M*(N5-1)
2040 FOR I= 1 TO M
2050 I4=I-M
2060 I6=N5-M
2070 I3=I3+1
2080 IF U(I)<=0 THEN 2170
2090 FOR J= 1 TO M
2100 I4=I4+M
2110 I6=I6+M
2120 IF U(J)<0 THEN 2150
2130 IF J-N5=0 THEN 2150
2140 D(I4)=D(I4)-D(I3)*D(I6)
2150 NEXT J
2160 D(I3)=D(I3)/(-R2)
2170 NEXT I
2180 R2=N-G(4)
2190 R2=D/R2
2200 H(10)=SQR(1.-(1.-H(6)*H(6))*R2)
2210 H(11)=H(8)*SQR(R2)
2220 REM TEST WHETHER THIS IS THE FIRST STEP
2230 IF G(4)-1>0 THEN2280
2240 PRINT "DEPENDENT VAR. IS";G(1)
2250 PRINT "NO. OF VAR. FORCED TO ENTER IN THE REG.";G(
2260 PRINT "NO. OF VAR. DELETED";G(3)
2270 GOSUB 2890
2280 PRINT "STEP NO.";G(4)
2290 PRINT "THE LAST VAR. ENTERED";G(5)
2300 IF G(4)-G(2)>0 THEN 2320
2310 PRINT "(FORCED VARIABLE)"
2320 REM
2330 PRINT "SUM OF SQUARES REDUCED";H(1)
2340 PRINT "PROPORTION REDUCED";H(2)
2360 PRINT "CUMULATIVE SS REDUCED";H(3)
2370 PRINT"CUMULATIVE PROPORTION REDUCED"H(4)" OF"H(5)
```

```
2400 PRINT"NUMBER OF VARIABLES ENTERED....."; G(4)
2410 PRINT"MULTIPLE CORRELATION COEFFICIENT";H(6)
2420 PRINT"ADJ. MULTIPLE CORRELATION COEFF.";H(10)
2430 PRINT"F-VALUE FOR ANOVA(FOR THE REG.) ";H(7)
2440 PRINT"STANDARD ERROR OF ESTIMATE......";H(8)
2450 PRINT"ADJUSTED STD. ERROR OF ESTIMATE ";H(11)
2460 GOSUB 2890
2470 PRINT "   VARIABLE     REGRESSION   STD.ERROR OF
     *  COMPUTED"
2480 PRINT "    NUMBER     COEFFICIENT    REG.COEFF.
     *  VALUE"
2490 N6=G(4)
2500 FOR I=1 TO N6
2510 PRINT L(I),P(I),S(I),T(I)
2520 NEXT I
2530 PRINT "INTERCEPT";H(9)
2540 GOSUB 2880
2550 N3=0
2560 IF N3>0 THEN 2580
2570 NEXT N4
2580 IF I2=0 THEN2640
2590 PRINT "EITHER THE MATRIX IS SINGULAR, OR THE RESIC
2600 PRINT "SS IS NEGATIVE IMPLYING EXTREME ILL CONDITI
2610 PRINT "SELECTION IGNORED"
2620 GOSUB 2880
2630 GOTO 770
2640 PRINT"ESTIMATED Y?  IF YES, TYPE 1; ELSE 0";
2650 INPUT K9
2670 IF K9=0 THEN 2840
2680RESTORE #1
2685 PRINT " "," "," 99% CONFIDENCE INTERVAL "
2690 PRINT "ID   P-HAT","P-OBS","LOWER LIMIT","UPPER LI
2700 K6=G(1)
2710 FOR I= 1 TO N
2720 Z9=H(9)
2730MAT READ #1, E
2731 K7=G(4)
2732 C4(1,1)=N
2733 FOR J1=1 TO K7
2734 J8=L(J1)+1
2735 J3=J1+1
2736 C4(1,J3)=C(1,J8)
2737 C4(J3,1)=C(1,J8)
2738 FOR J2=1 TO K7
2739 J4=J2+1
2740 K9=L(J2)+1
2741 C4(J3,J4)=C(J8,K9)
2742 C4(J4,J3)=C(J8,K9)
2743 NEXT J2
2744 NEXT J1
```

```
2745 MAT C1=INV(C4)
2746 A(1,1)=1
2750 FOR J=1 TO K7
2760 K8=L(J)
2761 J5=J+1
2762 A(1,J5)=E(K8)
2770 Z9=Z9+P(J)*E(K8)
2771 NEXT J
2777 MAT A1=TRN(A)
2778 MAT C2=A*C1
2779 MAT C3=C2*A1
2780 X3=H(11)*3.46*(SQR(C3(1,1)))
2781 X1=Z9-X3
2782 X1=(SIN(X1))**2
2783 X2=Z9+X3
2784 X2=(SIN(X2))**2
2790 Z8=E(K6)
2791 X9=(SIN(Z9))**2
2792 X8=(SIN(Z8))**2
2800 PRINT I;X9,X8,X1,X2
2810 NEXT I
2815 PRINT " THE (X'X) MATRIX  "
2816 MAT PRINT C4
2817 PRINT
2818 PRINT " THE INVERSE OF (X'X) "
2819 MAT PRINT C1
2820RESTORE #1
2830 GOSUB 2880
2840 PRINT"MORE SELECTION?  IF YES, TYPE 1; ELSE 0";
2850 INPUT N1
2860 IF N1>0 THEN 770
2870 STOP
2880 PRINT
2890 PRINT
2900 RETURN
2910 STOP
04000 DATA 78,15
04460 DATA 1,1,4,9,1,1,2,3,1,2,3,6,6,6,1.0472
04470 DATA 1,1,4,9,1,1,2,3,1,2,3,6,6,6,.9985
04480 DATA 1,1,4,9,1,1,2,3,1,2,3,6,6,6,.9343
04490 DATA 8,1,4,9,4,1,2,3,2,2,3,6,6,12,1.0866
04500 DATA 8,1,4,9,4,1,2,3,2,2,3,6,6,12,.9767
04510 DATA 8,1,4,9,4,1,2,3,2,2,3,6,6,12,1.0786
04520 DATA 27,1,4,9,9,1,2,3,3,2,3,6,6,18,.9839
04530 DATA 27,1,4,9,9,1,2,3,3,2,3,6,6,18,1.0095
04540 DATA 27,1,4,9,9,1,2,3,3,2,3,6,6,18,.9273
04640 DATA 1,1,9,4,1,1,3,2,1,3,2,6,6,6,1.1113
04650 DATA 1,1,9,4,1,1,3,2,1,3,2,6,6,6,1.1638
04660 DATA 1,1,9,4,1,1,3,2,1,3,2,6,6,6,1.1593
04670 DATA 8,1,9,4,4,1,3,2,2,3,2,6,6,12,1.1825
```

```
04680 DATA 8,1,9,4,4,1,3,2,2,3,2,6,6,12,1.1198
04690 DATA 8,1,9,4,4,1,3,2,2,3,2,6,6,12,1.0786
04700 DATA 27,1,9,4,9,1,3,2,3,3,2,6,6,18,.8962
04710 DATA 27,1,9,4,9,1,3,2,3,3,2,6,6,18,1.0169
04720 DATA 27,1,9,4,9,1,3,2,3,3,2,6,6,18,.9803
04730 DATA 1,1,9,9,1,1,3,3,1,3,3,9,9,9,.7921
04740 DATA 1,1,9,9,1,1,3,3,1,3,3,9,9,9,.7754
04750 DATA 1,1,9,9,1,1,3,3,1,3,3,9,9,9,.8827
04760 DATA 8,1,9,9,4,1,3,3,2,3,3,9,9,18,.7687
04770 DATA 8,1,9,9,4,1,3,3,2,3,3,9,9,18,.8288
04780 DATA 8,1,9,9,4,1,3,3,2,3,3,9,9,18,.8188
04790 DATA 27,1,9,9,9,1,3,3,3,3,3,9,9,27,.6983
04800 DATA 27,1,9,9,9,1,3,3,3,3,3,9,9,27,.7854
04810 DATA 27,1,9,9,9,1,3,3,3,3,3,9,9,27,.7118
05000 DATA 1,4,1,9,1,2,1,3,1,2,6,3,6,3,1.0244
05010 DATA 1,4,1,9,1,2,1,3,1,2,6,3,6,3,.9875
05020 DATA 1,4,1,9,1,2,1,3,1,2,6,3,6,3,1.0434
05030 DATA 8,4,1,9,4,2,1,3,2,2,6,3,6,6,1.0319
05040 DATA 8,4,1,9,4,2,1,3,2,2,6,3,6,6,1.0244
05050 DATA 8,4,1,9,4,2,1,3,2,2,6,3,6,6,1.0282
05060 DATA 27,4,1,9,9,2,1,3,3,2,6,3,6,9,.9660
05070 DATA 27,4,1,9,9,2,1,3,3,2,6,3,6,9,.9238
05080 DATA 27,4,1,9,9,2,1,3,3,2,6,3,6,9,.8963
05180 DATA 1,4,4,4,1,2,2,2,1,4,4,4,8,4,.8963
05190 DATA 1,4,4,4,1,2,2,2,1,4,4,4,8,4,.9238
05200 DATA 1,4,4,4,1,2,2,2,1,4,4,4,8,4,.9273
05210 DATA 8,4,4,4,4,2,2,2,2,4,4,4,8,8,.8455
05220 DATA 8,4,4,4,4,2,2,2,2,4,4,4,8,8,.9624
05230 DATA 8,4,4,4,4,2,2,2,2,4,4,4,8,8,.7219
05240 DATA 27,4,4,4,9,2,2,2,3,4,4,4,8,12,.6915
05250 DATA 27,4,4,4,9,2,2,2,3,4,4,4,8,12,.7185
05260 DATA 27,4,4,4,9,2,2,2,3,4,4,4,8,12,.6435
05360 DATA 1,4,9,1,1,2,3,1,1,6,2,3,6,3,1.2069
05370 DATA 1,4,9,1,1,2,3,1,1,6,2,3,6,3,1.1503
05380 DATA 1,4,9,1,1,2,3,1,1,6,2,3,6,3,1.1731
05390 DATA 8,4,9,1,4,2,3,1,2,6,2,3,6,6,1.1458
05400 DATA 8,4,9,1,4,2,3,1,2,6,2,3,6,6,1.0948
05410 DATA 8,4,9,1,4,2,3,1,2,6,2,3,6,6,1.1283
05420 DATA 27,4,9,1,9,2,3,1,3,6,2,3,6,9,.9624
05430 DATA 27,4,9,1,9,2,3,1,3,6,2,3,6,9,.9731
05440 DATA 27,4,9,1,9,2,3,1,3,6,2,3,6,9,.8759
05720 DATA 1,9,1,4,1,3,1,2,1,3,6,2,6,2,1.0862
05730 DATA 1,9,1,4,1,3,1,2,1,3,6,2,6,2,1.1198
05740 DATA 1,9,1,4,1,3,1,2,1,3,6,2,6,2,1.0907
05750 DATA 8,9,1,4,4,3,1,2,2,3,6,2,6,4,.9948
05760 DATA 8,9,1,4,4,3,1,2,2,3,6,2,6,4,1.0746
05770 DATA 8,9,1,4,4,3,1,2,2,3,6,2,6,4,1.1030
05780 DATA 27,9,1,4,9,3,1,2,3,3,6,2,6,6,.8556
05790 DATA 27,9,1,4,9,3,1,2,3,3,6,2,6,6,.8556
05800 DATA 27,9,1,4,9,3,1,2,3,3,6,2,6,6,.91
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05900 DATA 1,9,4,1,1,3,2,1,1,6,3,2,6,2,1.0511
05910 DATA 1,9,4,1,1,3,2,1,1,6,3,2,6,2,1.1283
05920 DATA 1,9,4,1,1,3,2,1,1,6,3,2,6,2,1.0989
05930 DATA 8,9,4,1,4,3,2,1,2,6,3,2,6,4,.9984
05940 DATA 8,9,4,1,4,3,2,1,2,6,3,2,6,4,1.0206
05950 DATA 8,9,4,1,4,3,2,1,2,6,3,2,6,4,1.0012
05960 DATA 27,9,4,1,9,3,2,1,3,6,3,2,6,6,.8288
05970 DATA 27,9,4,1,9,3,2,1,3,6,3,2,6,6,.8154
05980 DATA 27,9,4,1,9,3,2,1,3,6,3,2,6,6,.7954
06170 DATA 1,9,9,1,1,3,3,1,1,9,3,3,9,3,.6013
06180 DATA 1,9,9,1,1,3,3,1,1,9,3,3,9,3,.6365
06190 DATA 1,9,9,1,1,3,3,1,1,9,3,3,9,3,.6365
06200 DATA 8,9,9,1,4,3,3,1,2,9,3,3,9,6,.4801
06210 DATA 8,9,9,1,4,3,3,1,2,9,3,3,9,6,.5313
06220 DATA 8,9,9,1,4,3,3,1,2,9,3,3,9,6,.5120
6440 PRINT "K1","I","J"
6450 PRINT K1,I,J
6460 PRINT"B"
6470 MAT PRINT B
06480 Z1=ESM(X)
06490 Z2=ESL(X)
06500 PRINT"ESL=";Z2,"ESM=";Z1
06510 END
```

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